Water Science and Engineering, 2009, 2(4): 95-102
doi:10.3882/j.issn.1674-2370.2009.04.009

http://kkb.hhu.edu.cn
e-mail: wse@hhu.edu.cn

Mechanisms of hydraulic fracturing in cohesive soil

Jun-jie WANG*[1, 2], Hui-ping ZHANG[1, 2], Ming-jie ZHAO[1, 2], Xin LIN[1, 2]

1. School of River and Ocean Engineering, Chongqing Jiaotong University, Chongqing 400074, P. R. China
2. Key Laboratory of Hydraulic and Waterway Engineering of Ministry of Education, Chongqing Jiaotong University, Chongqing 400074, P. R. China

Abstract: Hydraulic fracturing in the soil core of earth-rockfill dams is a common problem affecting the safety of the dams. Based on fracture tests, a new criterion for hydraulic fracturing in cohesive soil was suggested. Using this criterion, the mechanisms of hydraulic fracturing in cubic soil specimens were investigated. The results indicate that the propagation of the crack in a cubic specimen under water pressure occurs in a mixed mode I-II if the crack face is not perpendicular to any of the principal stresses, and the crack most likely to propagate is the one that is perpendicular to the minor principal stress and propagates in mode I.

Key words: *hydraulic fracturing; cohesive soil; crack; propagation; mixed mode I-II*

1 Introduction

Statistical data assembled by ICOLD (1983, 1995) and statistical analyses by Foster et al. (2000) indicate that approximately 30% to 50% of earth dam failures can be attributed to progressive piping and erosion. Progressive failures and/or cracks in earth dams that have been reported and investigated include those of the Hyttejuvet Dam, by Kjaernsli and Torblaa (1968), the Teton Dam, by Seed and Duncan (1981), and several old British dams, by Dounias et al. (1996). For earth-rockfill dams with soil cores, the progressive piping and erosion may result in concentrated leakage of reservoir water through the cores.

A great number of high earth-rockfill dams with soil cores are under construction or will soon be constructed in Western China, where water resources are abundant. These include the Nuozhadu Dam (261.5 m in height) on the Lancang River in Yunnan Province, and the Shuangjiangkou Dam (322 m in height) and the Changhe Dam (240 m in height) on the Dadu River in Sichuan Province. The core soils of earth-rockfill dams may be subject to cracks, which result from the arching action and/or hydraulic fracturing (Zhu and Wang 2004). The engineers should decide whether the cracks are likely to extend and affect the integrity of the structure or whether they are stable and self-healing.

This work was supported by the National Natural Science Foundation of China (Grant No. 50779081), the Key Project of the Chinese Ministry of Education (Grant No. 208114), and the Science and Technology Project of the Chongqing Municipal Education Commission of China (Grant No. KJ080428).
*Corresponding author (e-mail: *wangjunjiehhu@163.com*)
Received Sep. 2, 2009; accepted Nov. 10, 2009

Core cracks are induced by many factors, and the cores of most earth dams may contain some cracks. If the core cracks are propagating due to a change in stress states or other factors, the safety of the dams will be affected. The propagation and/or initiation of the core cracks under water pressure is called hydraulic fracturing in dam engineering (Sherard 1986). The problem of hydraulic fracturing in the soil cores of earth dams has been regarded as a very important geotechnical problem related to the safety of the dams since the failure of the Teton Dam on June 5, 1976 (IPRCTDF 1976; Seed et al. 1976; USDITDFRG 1977). In the last three decades, the problem of hydraulic fracturing has received wide attention from many investigators, including Kulhawy and Gurtowski (1976), Jaworski et al. (1981), Mori and Tamura (1987), Lo and Kaniaru (1990), Yanagisawa and Panah (1994), Andersen et al. (1994), Ng and Small (1999), and Wang and Zhu (2007). However, the problem of hydraulic fracturing in soil cores of earth dams is still far from solved, especially for earth-rockfill dams with heights of 200 m to 300 m, such as those under or soon-to-be under construction in Western China.

This study suggests a new criterion for hydraulic fracturing in cohesive soil. Using that criterion, the mechanisms of hydraulic fracturing in a cubic cohesive soil specimen were investigated.

2 Criterion for hydraulic fracturing

Previous studies have suggested different methods for determining the water pressure required to induce hydraulic fracturing. These methods may be classified into three groups (Wang and Zhu 2006). The first are theoretical methods such as the cylindrical or spherical cavity expansion theories in elastic or elastic-plastic mechanics (as in Yanagisawa and Panah (1994)). The second are empirical methods based on field or laboratory tests (such as those of Mori and Tamura (1987)). The last are conceptual models based on laboratory tests and theories in fracture mechanics (FM) (as in Murdoch (1993c)).

A crack in the core, which allows water to enter the core, is a prerequisite for hydraulic fracturing (Wang and Zhu 2007). Thus, hydraulic fracturing is actually the propagation of the crack under water pressure. FM can be used to investigate the problem.

The finite element method (FEM) has been widely used in simulating stresses and strains on earth-rockfill dams during construction and impounding. This should be considered while establishing the criterion for hydraulic fracturing. The earth-rockfill dam is usually simplified as a plane strain problem in the FEM analysis. Thus, the criterion for hydraulic fracturing should also be established based on the plane strain condition.

Under the plane strain condition, the crack propagation may be in mode I, mode II, or a mixed mode I-II. Because the stress state in the core is very complex and the spreading of the crack can be induced by the combination of normal stress perpendicular to the crack face and shear stress parallel to the crack face (Vallejo 1993), the criterion for hydraulic fracturing should be investigated according to the mixed mode I-II.

Based on experimental study of the fracture behavior of a silty clay that is the core material of the Nuozhadu Earth-Rockfill Dam in Western China (Wang et al. 2007), a criterion for hydraulic fracturing was formulated:

$$\left(K_{\mathrm{I}}^2+K_{\mathrm{II}}^2\right)^{0.5}=K_{\mathrm{IC}} \tag{1}$$

where K_{IC} is the mode I fracture toughness of the core soil, and K_{I} and K_{II} are the stress intensity factors of mode I and mode II cracks, respectively.

The J integral proposed by Rice (1968) is a parameter indicating the intensity of nominal stress, and it is a constant for different integral routes. The relationship between the J integral and stress intensity factor for a mixed mode I-II crack under plane strain conditions can be described as (Anderson 1991)

$$J=\frac{1-\nu^2}{E}\left(K_{\mathrm{I}}^2+K_{\mathrm{II}}^2\right) \tag{2}$$

where E and ν are the Young's modulus and the Poisson's ratio of the material, respectively. The value of J can be obtained with the FEM (Hellen 1975; Delorenzi 1985; Hamoush and Salami 1993). The value of $\left(K_{\mathrm{I}}^2+K_{\mathrm{II}}^2\right)^{0.5}$ in Eq. (1) can be obtained from Eq. (2).

3 Hydraulic fracturing in cubic soil

A cubic specimen with an envelope-shaped crack was used to investigate the problem of hydraulic fracturing in soil (Murdoch 1993a). In his tests, the author considered the crack face to be perpendicular to the minor principal stress, and the water pressure that induced hydraulic fracturing was applied through a very thin pipe inserted in the crack along its centerline. In this condition, only the normal stress was applied on the crack face. In more common cases, the crack face should not be perpendicular to any of the principal stresses, such as in the case shown in Fig. 1(a). It can be simplified as the plane strain crack shown in Fig. 1(b). The normal stress σ_n and shear stress σ_t on the crack face in the figure can be expressed as

$$\sigma_n=\frac{1}{2}\left[\sigma_y+\sigma_x+\left(\sigma_y-\sigma_x\right)\cos 2\beta\right] \tag{3}$$

$$\sigma_t=\frac{1}{2}\left(\sigma_y-\sigma_x\right)\sin 2\beta \tag{4}$$

where σ_x, σ_y, and σ_z are the normal stresses on the surfaces of the cubic specimen in the x, y, and z directions, respectively, and β is the angle of the crack face slope.

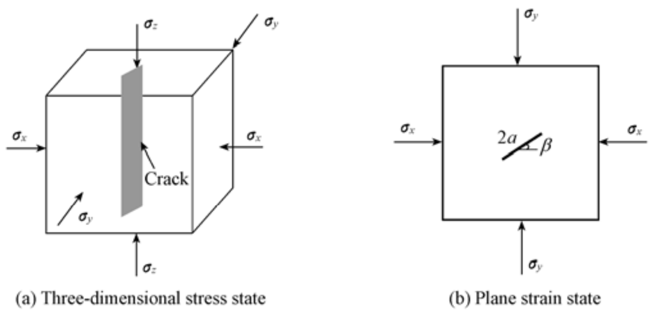

Fig. 1 Cubic specimen with crack for hydraulic fracturing test (2a is the length of the crack)

When 0º < β < 90º, the propagation of the crack occurs in a mixed mode I-II. Stress intensity factors K_{I} and K_{II} at the tip of the crack can be obtained by the following equation (Anderson 1991):

$$K_{\mathrm{I}} = -\sigma\sqrt{\pi a} \tag{5}$$

$$K_{\mathrm{II}} = \tau\sqrt{\pi a} \tag{6}$$

where σ and τ are the effective normal stress and effective shear stress on the crack face, respectively. When σ is compression stress, the value of K_{I} in Eq. (5) is negative.

In testing, water pressure exerted on the crack face may induce water wedging. To simplify the analysis, it is assumed that the intensity of the water wedging is equal to that of the water pressure. The intensity of the water pressure can be expressed as

$$p = \rho g H \tag{7}$$

where p is the intensity of water pressure, ρ is the density of water, g is the acceleration of gravity, and H is the hydraulic head in the crack.

3.1 Calculation of K_{I}

The effective normal stress on the crack face in a hydraulic fracturing test can be expressed as

$$\sigma = \sigma_n - p = \frac{1}{2}\left[\sigma_y + \sigma_x + \left(\sigma_y - \sigma_x\right)\cos 2\beta\right] - \rho g H \tag{8}$$

Substituting Eq. (8) into Eq. (5), K_{I} is calculated as

$$K_{\mathrm{I}} = -\left\{\frac{1}{2}\left[\sigma_y + \sigma_x + \left(\sigma_y - \sigma_x\right)\cos 2\beta\right] - \rho g H\right\}\sqrt{\pi a} \tag{9}$$

3.2 Calculation of K_{II}

3.2.1 Case of open crack

For the case of an open crack, when the shear strength of the crack itself can be neglected, K_{II} can be obtained by substituting Eq. (4) into Eq. (6), as follows:

$$K_{\mathrm{II}} = \frac{1}{2}\left(\sigma_y - \sigma_x\right)\sin 2\beta\sqrt{\pi a} \tag{10}$$

3.2.2 Case of closed crack

For the case of a closed crack, when the shear strength of the crack itself cannot be neglected, the effective shear stress on the crack face can be obtained from the following equation:

$$\tau = \sigma_t - \tau^* \tag{11}$$

where τ^* is the shear stress induced by the resistance of the crack to shear deformation, called reverse shear stress here because of its opposite direction to σ_t . The expression of τ^* is

$$\begin{cases} \tau^* = \sigma_t & \sigma \geq 0\,,\ \text{and}\ \sigma_t \leq \tau_{\mathrm{f}} \\ \tau^* = \tau_{\mathrm{f}} & \sigma \geq 0\,,\ \text{and}\ \sigma_t > \tau_{\mathrm{f}} \\ \tau^* = 0 & \sigma < 0 \end{cases} \tag{12}$$

where τ_{f} is the shear strength of the crack, obtained from the Mohr-Coulomb theory of strength.

The shear strength of the crack decreases as water enters. This paper expresses cohesion and the internal friction angle of the crack before water enters as c_1 and φ_1, respectively, and expresses those values after water enters as c_2 and φ_2. The value of τ_{f} can be obtained from Eq. (13):

$$\tau_{\mathrm{f}} = \sigma \tan\varphi_2 + c_2 \tag{13}$$

Combining Eqs. (11), (12), and (13), the effective shear stress τ can be expressed as

$$\tau = \begin{cases} 0 & \sigma \geq 0, \text{ and } \sigma_t \leq \sigma\tan\varphi_2 + c_2 \\ \sigma_t - \sigma\tan\varphi_2 - c_2 & \sigma \geq 0, \text{ and } \sigma_t > \sigma\tan\varphi_2 + c_2 \\ \sigma_t & \sigma < 0 \end{cases} \tag{14}$$

where σ_t and σ can be obtained from Eqs. (4) and (8), respectively.

Substituting Eqs. (4), (8), and (14) into Eq. (6), K_{II} can be expressed as

$$K_{\mathrm{II}} = \begin{cases} 0 & H \leq H_1 \\ \left\{0.5\left(\sigma_y - \sigma_x\right)\sin 2\beta - \left[0.5\left(\sigma_y + \sigma_x\right) + 0.5\left(\sigma_y - \right.\right.\right. & \\ \quad \left.\left.\left.\sigma_x\right)\cos 2\beta - \rho g H\right]\tan\varphi_2 - c_2\right\}\sqrt{\pi a} & H_1 < H \leq H_2 \\ \dfrac{1}{2}\left(\sigma_y - \sigma_x\right)\sin 2\beta\sqrt{\pi a} & H > H_2 \end{cases} \tag{15}$$

where

$$H_1 = \frac{\frac{1}{2}\left[\sigma_y + \sigma_x + \left(\sigma_y - \sigma_x\right)\cos 2\beta\right]}{\rho g} - \frac{\frac{1}{2}\left(\sigma_y - \sigma_x\right)\sin 2\beta - c_2}{\rho g\tan\varphi_2} \tag{16}$$

$$H_2 = \frac{\frac{1}{2}\left[\sigma_y + \sigma_x + \left(\sigma_y - \sigma_x\right)\cos 2\beta\right]}{\rho g} \tag{17}$$

If $H_1 \geq H_2$, H_1 is considered to be the same as H_2.

3.3 Calculation of $\left(\boldsymbol{K}_{\mathrm{I}}^2 + \boldsymbol{K}_{\mathrm{II}}^2\right)^{0.5}$

For the case of an open crack, the value $\left(K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2\right)^{0.5}$, which is used to estimate hydraulic fracturing, can be obtained by combining Eqs. (9) and (10). For the case of a closed crack, the value $\left(K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2\right)^{0.5}$ can be obtained by combining Eqs. (9) and (15).

Fig. 2 shows all the stress intensity factors discussed in the previous paragraphs. K_{I} has a linear relationship with hydraulic head, which may be explained by the fact that K_{I} is not affected by the shear strength of the crack. When the hydraulic head increases, K_{I} changes from negative to positive values. It is equal to zero at hydraulic head H_0 because the effective normal stress on the crack face equals zero. Therefore, the propagation of the crack may be mode II at hydraulic head H_0. When the hydraulic head is greater than H_0, the effective normal stress is tensile stress, and hydraulic fracturing may be induced. The crack shear strength influences K_{II} only at a hydraulic head less than H_0. The influence of the crack shear strength on $\left(K_{\mathrm{I}}^{2}+K_{\mathrm{II}}^{2}\right)^{0.5}$ becomes negligible at a hydraulic head greater than H_0. In this case, investigation of the hydraulic fracturing in laboratory tests shows that the crack shear strength has no influence.

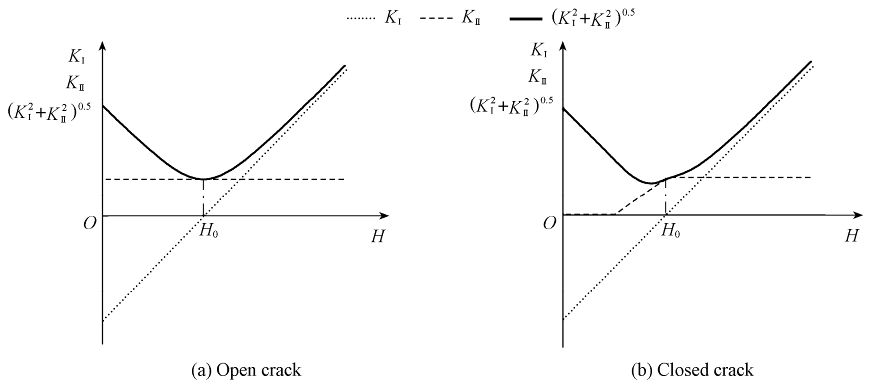

Fig. 2 Stress intensity factors at crack tip in hydraulic fracturing test sample (β=60º)

3.4 Dangerous crack angle

Given that the stress state in Fig. 1(b) is $\sigma_y = 2\sigma_x$, the value of $\left(K_{\mathrm{I}}^{2}+K_{\mathrm{II}}^{2}\right)^{0.5}$ for different values of included angle β in the case of an open crack can be obtained (Fig. 3). The crack obtained at $\beta = 90°$ may propagate first because it has the maximum value of $\left(K_{\mathrm{I}}^{2}+K_{\mathrm{II}}^{2}\right)^{0.5}$ when the hydraulic head is greater than H_0. The propagation of the crack at $\beta = 90°$ may occur in mode I because K_{II} is equal to zero. This accords with the work of Murdoch (1993b).

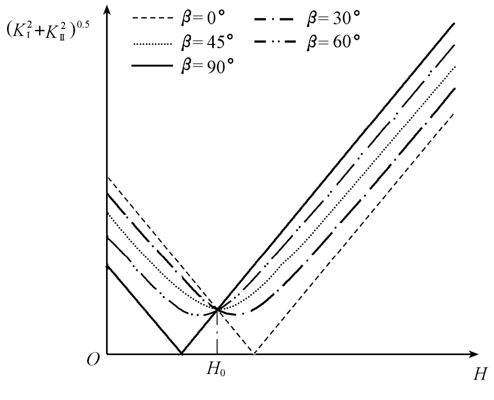

Fig. 3 Variation of $\left(K_{\mathrm{I}}^{2}+K_{\mathrm{II}}^{2}\right)^{0.5}$ with hydraulic head for different values of included angle β

4 Conclusions

The problem of hydraulic fracturing in the soil cores of high earth-rockfill dams is a common and important geotechnical and hydraulic problem affecting the safety of the dams. In the last three decades, this problem has received a large amount of attention from many researchers, but further study is still necessary. The reasonableness of the criterion suggested in this paper was theoretically verified by analyzing the hydraulic fracturing in a cubic soil specimen, but it is still necessary to verify it further in laboratory experiments. The propagation of the crack in a cubic soil specimen under water pressure occurs in a mixed mode I-II if the crack face is not perpendicular to any of the principal stresses, and the crack that propagates most easily is the one perpendicular to the minor principal stress and propagates in mode I.

References

Andersen, K. H., Rawlings, C. G., Lunne, T. A., and By, T. H. 1994. Estimation of hydraulic fracture pressure in clay. *Canadian Geotechnical Journal*, 31(6), 817-828. [doi:10.1139/t94-099]

Anderson, T. L. 1991. *Fracture Mechanics: Fundamentals and Applications* (1st edition). Orlando: CRC Press.

Delorenzi, H. G. 1985. Energy release rate calculations by the finite element. *Engineering Fracture Mechanics*, 21(1), 129-143.

Dounias, G. T., Potts, D. M., and Vaughan, P. R. 1996. Analysis of progressive failure and cracking in old British dams. *Geotechnique*, 46(4), 621-640.

Foster, M., Fell, R., and Spannagle, M. 2000. The statistics of embankment dam failures and accidents. *Canadian Geotechnical Journal*, 37(5), 1000-1024. [doi:10.1139/cgj-37-5-1000]

Hamoush, S. A., and Salami, M. R. 1993. A stiffness derivative technique to determine mixed mode stress intensity factors of rectilinear anisotropic solids. *Engineering Fracture Mechanics*, 44(2), 297-305.

Hellen, T. K. 1975. On the method of virtual crack extensions. *International Journal for Numerical Methods in Engineering*, 9(1), 187-207. [doi:10.1002/nme.1620090114]

Independent Panel to Review Cause of Teton Dam Failure (IPRCTDF). 1976. *Report to U. S. Department of the Interior and the State of Idaho on Failure of Teton Dam.* Denver: U. S. Bureau of Reclamation.

International Commission on Large Dams (ICOLD). 1983. *Deterioration of Dams and Reservoirs: Examples and Their Analysis*. Paris: International Commission on Large Dams.

International Commission on Large Dams (ICOLD). 1995. *Dam Failures Statistical Analysis, Bulletin 99*. Paris: International Commission on Large Dams.

Jaworski, G. W., Seed, H. B., and Duncan, J. M. 1981. Laboratory study of hydraulic fracturing. *Journal of the Geotechnical Engineering Division*, 107(6), 713-732.

Kjaernsli, B., and Torblaa, I. 1968. *Leakage through Horizontal Cracks in the Core of Hyttejuvet Dam.* Oslo: Norwegian Geotechnical Institute.

Kulhawy, F. H., and Gurtowski, T. M. 1976. Load transfer and hydraulic fracturing in zoned dams. *Journal of the Geotechnical Engineering Division*, 102(9), 963-974.

Lo, K. Y., and Kaniaru, K. 1990. Hydraulic fracture in earth and rock-fill dams. *Canadian Geotechnical Journal*, 27(4), 496-506.

Mori, A., and Tamura, M. 1987. Hydrofracturing pressure of cohesive soils. *Soils and Foundations*, 27(1), 14-22.

Murdoch, L. C. 1993a. Hydraulic fracturing of soil during laboratory experiments, Part 1. Methods and observations. *Geotechnique*, 43(2), 255-266. [doi:10.1680/geot.1993.43.2.255]

Murdoch, L. C. 1993b. Hydraulic fracturing of soil during laboratory experiments, Part 2. Propagation. *Geotechnique*, 43(2), 267-276. [doi:10.1680/geot.1993.43.2.267]

Murdoch, L. C. 1993c. Hydraulic fracturing of soil during laboratory experiments, Part 3. Theoretical analysis. *Geotechnique*, 43(2), 277-287. [doi:10.1680/geot.1993.43.2.277]

Ng, A. K. L., and Small, J. C. 1999. A case study of hydraulic fracturing using finite element methods. *Canadian Geotechnical Journal*, 36(5), 861-875. [doi:10.1139/cgj-36-5-861]

Rice, J. R. 1968. A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, 35(2), 379-386.

Seed, H. B., Leps, T. M., Duncan, J. M., and Bieber, R. E. 1976. Hydraulic fracturing and its possible role in the Teton Dam failure. *Appendix D of Report to U. S. Department of the Interior and State of Idaho on Failure* *of Teton Dam*. Denver: U. S. Bureau of Reclamation.

Seed, H. B., and Duncan, J. M. 1981. The Teton dam failure: A retrospective review. *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering*, 4, 219-238. Rotterdam: Balkema.

Sherard, J. L. 1986. Hydraulic fracturing in embankment dams. *Journal of Geotechnical Engineering*, 112(10), 905-927. [doi:10.1061/(ASCE)0733-9410(1986)112:10(905)]

U. S. Department of the Interior Teton Dam Failure Review Group (USDITDFRG). 1977. *Failure of Teton Dam: A Report of Findings.* Washington, D. C.: U. S. Department of the Interior Teton Dam Failure Review Group.

Vallejo, L. E. 1993. Shear stresses and the hydraulic fracturing of earth dam soils. *Soils and Foundations*, 33(3), 14-27.

Wang, J. J., and Zhu, J. G. 2006. Review on computing theories of hydraulic fracturing in soil. *Proceedings of the 2nd National Symposium on Geotechnical Engineering of China*, 2, 231-237. Wuhan: Science Press. (in Chinese)

Wang, J. J., and Zhu, J. G. 2007. Numerical study on hydraulic fracturing in the core of an earth rockfill dam. *Dam Engineering*, XVII(4), 271-293.

Wang, J. J, Zhu, J. G., Chiu, C. F., and Chai, H. J. 2007. Experimental study on fracture behavior of a silty clay. *Geotechnical Testing Journal*, 30(4), 303-311. [doi:10.1520/GTJ100715]

Yanagisawa, E., and Panah, A. K. 1994. Two dimensional study of hydraulic fracturing criteria in cohesive soils. *Soils and Foundations*, 34(1), 1-9.

Zhu, J. G., and Wang, J. J. 2004. Investigation on arching action and hydraulic fracturing of core rockfill dam. *Proceedings of the 4th International Conference on Dam Engineering*, 1171-1180. Rotterdam: Balkema.