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Analytical study on water hammer pressure in pressurized conduits with a throttled surge chamber for slow closure

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Abstract: This paper presents an analytical investigation of water hammer in a hydraulic pressurized pipe system with a throttled surge chamber located at the junction between a tunnel and a penstock, and a valve positioned at the downstream end of the penstock. Analytical formulas of maximum water hammer pressures at the downstream end of the tunnel and the valve were derived for a system subjected to linear and slow valve closure. The analytical results were then compared with numerical ones obtained using the method of characteristics. There is agreement between them. The formulas can be applied to estimating water hammer pressure at the valve and transmission of water hammer pressure through the surge chamber at the junction for a hydraulic pipe system with a surge chamber.

Key words: *throttled surge chamber; water hammer; orifice; analytical formula*

1 Introduction

Since the provision of a surge chamber (also referred to as a surge shaft or surge tank) in a pressurized pipe system can transform rapid flow change generated by closing or opening a valve/turbine into mass oscillation in the chamber, and lead to the reduction of water hammer pressure, the hydraulic characteristics of this arrangement have been extensively studied experimentally and theoretically (Jaeger 1977; Chaudhry 1987; Zhang and Liu 1992). These studies cover various types of surge chambers, including simple, throttled (orifice), differential, one-way, and air cushion (closed) surge chambers. Much effort so far has been devoted to the behavior of water hammer generated at the valve or turbine using analytical methods for pipe systems with surge chambers. A comprehensive review of analytical studies on pressure transmission in pipe systems has been conducted (Almeida and Koelle 1992).

Earlier analytical studies (Allievi 1913) used transient flow theory to derive formulas for water hammer in a simple pipe of constant diameter and wall thickness with a very large reservoir located upstream and a valve positioned downstream. These studies have been extended to piping systems with throttled surge chambers or other types of surge chambers,

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such as differential or air cushion chambers, and transmission pressure through the chamber in the case of rapid or instantaneous valve closure has been investigated analytically (Jaeger 1933; Zienkiewicz and Hawkins 1954; Shima and Hino 1960; Mosonyi and Seth 1975; Wang and Ma 1986; Zhang and Liu 1992; Ma 1996). These analytical formulas provide a good design/analysis tool for engineers and researchers, particularly in the hydropower industry. All of these studies are based on the assumption that

$$T_s + 2L_2/a_2 \le 2L_1/a_1 + 2L_2/a_2$$

where T_s is the valve closure time; L_1 and L_2 are the length between the water surface and the tunnel-penstock-chamber junction and the penstock length, respectively; and a_1 and a_2 are the wave speeds in the chamber and the penstock, respectively. However, the valve/turbine actually does not close instantaneously, and its normal closing time T_S (≈ 10 s) is at least ten times longer than $2L_1/a_1$ (< 1 s). Therefore, the aforementioned assumption is not valid and the analytical formula is not applicable to actual hydroelectric pipe systems. This is evident from graphical (Escande 1949; Zienkiewicz and Hawkins 1954; Shima and Hino 1960), numerical (Peng and Yang 1986; Prenner and Drobir 1997) and experimental (Shima and Hino 1960; Bernhart 1975; Peng and Yang 1986; Wang and Yang 1989; Prenner and Drobir 1997) investigation of transmission pressure in many hydropower plants with slow valve/turbine closure. To the authors' knowledge, no analytical formula has been reported for transmission pressures for the case of slow closure of the turbine/valve.

This paper presents analytical formulas of maximum water hammer pressures at the downstream end of the tunnel and the valve, for a hydraulic pressurized pipe system with a throttled surge chamber subjected to linear and slow valve closure. In the system, a throttled surge chamber is located at the junction between a tunnel and a penstock, and a valve is positioned at the downstream end of the penstock. The analytical results are then compared with numerical ones obtained using the method of characteristics to demonstrate the validity of the formulas.

2 Mathematical model

2.1 Water hammer equations

Consider a hydraulic system consisting of a diversion tunnel, a surge chamber, and a penstock, as shown in Fig. 1. The fluid is described by the piezometric head $H(x, t)$ and cross-sectional average velocity $V(x, t)$, where x is the spatial coordinate along the pipeline and t is the temporal coordinate. In this study, the friction loss was assumed to be small and was therefore neglected. The equations for water hammer are

$$\frac{\partial V}{\partial t} + g\frac{\partial H}{\partial x} = 0 \tag{1}$$

$$\frac{g}{a^2}\frac{\partial H}{\partial t} + \frac{\partial V}{\partial x} = 0 \tag{2}$$

where a is the wave speed and g is the gravitational acceleration. The general integrals of the two simultaneous partial differential equations above are

$$H - H_0 = \phi\left(t - x/a\right) + \psi\left(t + x/a\right) \tag{3}$$

$$V - V_0 = -\frac{g}{a}\left[\phi\left(t - x/a\right) - \psi\left(t + x/a\right)\right] \tag{4}$$

where ϕ is a function that can be interpreted as a wave moving in the $+x$ direction, and ψ is a function that can be interpreted as a wave moving in the $-x$ direction; H_0 is the initial piezometric head; and V_0 is the initial cross-sectional average velocity in the tunnel. The exact forms of functions ϕ and ψ depend on particular boundary conditions.

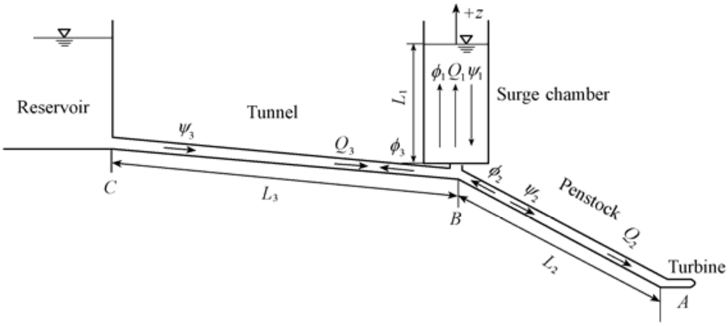

Fig. 1 Sketch of pressurized pipe system with throttled surge chamber

(ϕ_1 and ϕ_3 are the transmitted waves propagating to the surge chamber and the tunnel, respectively; ϕ_2 is the water hammer wave generated at the valve/turbine propagating upstream along the penstock upon closure of the valve/turbine; ψ_1, ψ_2, and ψ_3 are the reflected waves at the free water surface in the chamber, the junction, and the inlet of the tunnel, respectively; Q_1, Q_2, and Q_3 are the flow rates to the surge chamber, the penstock, and the tunnel, respectively; L_3 is the length of the upstream tunnel.)

2.2 Tunnel-surge chamber-penstock joint equation

The general assumptions made in the course of the analysis of the junction of the tunnel, the chamber, and the penstock (Fig. 2) are as follows:

(1) the continuity equation is valid for the system;

(2) at any instant there are identical pressure heads at points B_0, B_1, B_2, and B_3, which denote the entrance to the surge chamber just above the orifice, the entrance to the surge chamber just below the orifice, the entrance to the penstock, and the entrance to the tunnel, respectively;

(3) the velocity is uniformly distributed across each conduit at the junction;

(4) incident and reflected pressure waves from the junction are plane-fronted;

(5) at any instant the difference of pressure on each side of the orifice is equal to the water head loss corresponding to the flow under steady conditions;

(6) the inertia force of the water column and the friction in the surge chamber are small and can be neglected;

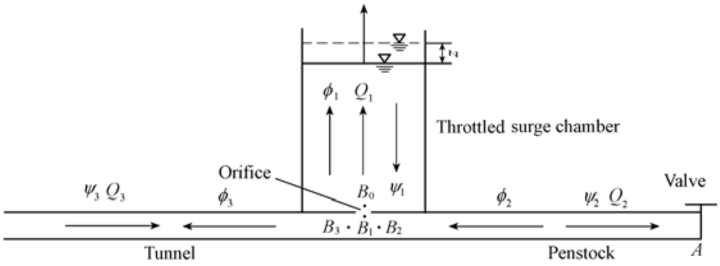

Fig. 2 Tunnel-surge tank-penstock joint

(7) the upstream tunnel is long enough that, upon the full closure of the valve, the reflected wave ψ_3 does not arrive at the junction.

The conditions at the junction can be expressed as

$$H_2(t)=H_3(t)=H_1(t)=H_0(t)+kQ_1(t)\left|Q_1(t)\right|=H_2(0)+z+kQ_1(t)\left|Q_1(t)\right| \tag{5}$$

$$Q_3(t)=Q_1(t)+Q_2(t) \tag{6}$$

$$z(t)=\frac{1}{f_1}\int_0^t Q_1(t)\,\mathrm{d}t \tag{7}$$

For steady flow, they can be expressed as

$$\begin{cases} H_1(0)=H_2(0)=H_3(0)=H_0 \\ Q_2(0)=Q_3(0)=Q_0 \\ Q_1(0)=0 \end{cases} \tag{8}$$

where H_0 is the initial pressure head at the entrance to the surge chamber just above the orifice; Q_0 is the initial flow rate in the tunnel; $H_0(t)$ is the pressure head at the entrance to the surge chamber just above the orifice at time *t*; $H_1(t)$ and $Q_1(t)$ are the pressure head and the flow rate at the entrance to the surge chamber just below the orifice at time *t*, respectively; $H_2(t)$ and $Q_2(t)$ are the pressure head and the flow rate at the entrance to the penstock, respectively; $H_3(t)$ and $Q_3(t)$ are the pressure head and the flow rate at the downstream end of the tunnel, respectively; *z* is the increment of water level in the surge chamber; A_s is the cross-sectional area of the surge chamber; and *k* is the coefficient defined as follows by Zienkiewicz and Hawkins (1954):

$$k=\frac{1}{2g}\left(\frac{1}{Cf_0}-\frac{1}{A_s}\right)^2 \tag{9}$$

where *C* is the contraction coefficient of the orifice and f_0 is the area of the orifice.

In this study we considered linear valve closure and introduced a set of dimensionless quantities of relative water hammers at the valve and the entrance of the penstock, defined as follows:

$$\xi(t)=\left[H_A(t)-H_A(0)\right]/H_A(0),\quad h_\mathrm{p}(t)=\left[H_2(t)-H_2(0)\right]/H_2(0) \tag{10}$$

where $\xi(t)$ and $h_\mathrm{p}(t)$ are the relative water hammers at the valve and the entrance of the penstock at time *t*, respectively; $H_A(t)$ is the pressure head at the valve at time *t*; and $H_A(0)$

is the initial pressure head at the valve. Allievi (1913) found that when $\tau_0\mu > 1$, where τ_0 is the initial degree of valve opening and $\mu = (a_2 Q_0)/(2gH_0 f_2)$, with f_2 being the cross-sectional area of the penstock, the maximum value of ξ occurs at $t = T_s$. This is also referred to as final water hammer. Most hydropower plants satisfy the condition of $\tau_0\mu > 1$, but there are some very high-head power plants (≥ 300 m) that do not. Upon full closure of the valve, the pressure head at the valve begins to decrease and then oscillate in the period of 2θ, as illustrated in Fig. 3 (the area of the orifice is 22.6195 m^2), and a decompression wave begins to move upstream. The pressure head at the junction begins to oscillate at $t = T_s + \theta/2$, as illustrated in Fig. 4. The maximum relative water hammers $\xi(t)$ and $h_p(t)$ occur at time instants $T_s + \theta/2$ and T_s, respectively, i.e., $h_p^* = h_p(T_s + \theta/2)$ and $\xi^* = \xi(T_s)$. It should be mentioned that for a hydroelectric power plant with a very long diversion tunnel, the amplitude of mass oscillation Z^* in the surge chamber can be estimated as follows (Jaeger 1933):

$$Z^* = Q_0 \sqrt{\frac{L_3}{gf_3 A_s}} \tag{11}$$

where f_3 is the cross-sectional area of the tunnel. It is indicated in Eq. (11) that mass oscillation Z^* will exceed $H_A(0)\xi^*$ and $H_2(0)h_p^*$ for a certain range of L_3, so that ξ^* and h_p^* might not be the largest in this condition. The second peak of the water hammer wave in Fig. 4 is larger than the first peak, and the value of $H_2(t)$ at time $T_s + n\theta/2$ is larger than that at time $T_s + (n-1)\theta/2$. This is due to the fact that the increment of the water level in the surge chamber exceeds the decrement of transmission pressure. Fig. 5 shows the variation of the increment of the water level subtracted from the pressure head at the entrance of the penstock with time. It can be seen from Fig. 5 that the value $H_2(t) - H_2(0) - z$ increases significantly with time, reaches its maximum at time $T_s + \theta/2$, and then oscillates. The peak value of $H_2(t) - H_2(0) - z$ decreases slowly after time $T_s + \theta/2$. This shows that the maximum transmission water hammer is reached at time $T_s + \theta/2$.

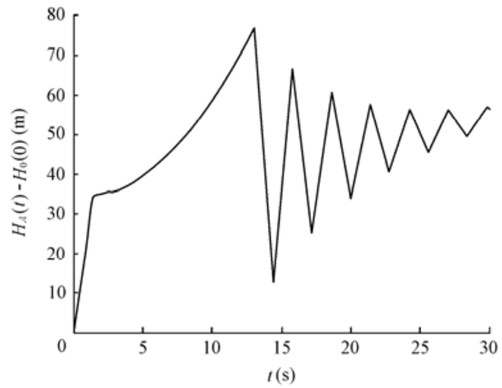

Fig. 3 Pressure head at valve

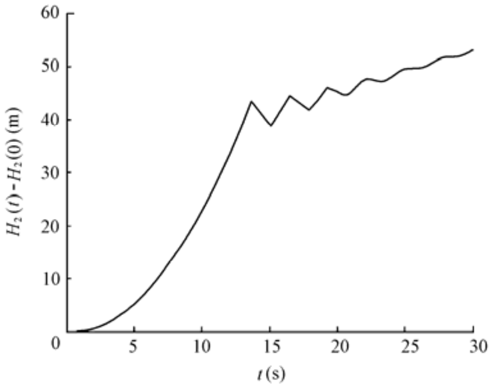

Fig. 4 Pressure head at bottom of surge chamber

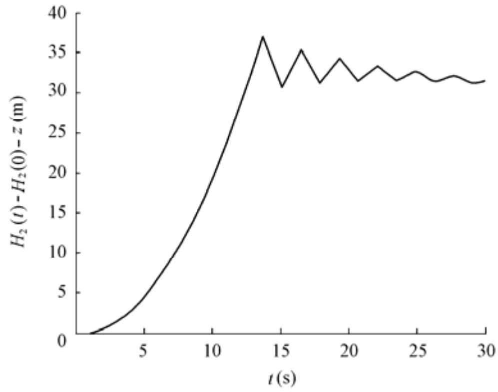

Fig. 5 Transmitted pressure minus increment of water level in chamber

The period of mass oscillation in the surge chamber (100 s to 500 s) is much larger than the time taken for valve closure T_s (10 s), so it is in the time interval $0 \le t \le \left(T_s + \theta/2\right)$, with $Q_1\left(t\right) \ge 0$, as shown in Fig. 6. Therefore, at $t = T_s + \theta/2$ the absolute value sign in Eq. (5) can be removed.

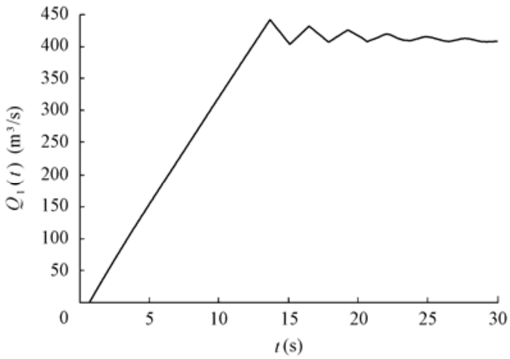

Fig. 6 Flow rate into surge chamber

Provided that the diversion tunnel is long enough, the reflected pressure wave ψ_3 has not yet arrived at the junction when the valve has just fully closed. This yields

$$\psi_3 = 0 \tag{12}$$

Substituting Eq. (12) into Eqs. (3) and (4), the following equations are obtained:

$$H_3\left(t\right) - H_3\left(0\right) = \phi_3 \tag{13}$$

$$Q_3\left(t\right) - Q_3\left(0\right) = u_3\left(-\phi_3\right) \tag{14}$$

where $u_3 = gf_3/a_3$; a_3 is the wave speed in the tunnel.

Combining Eqs. (13) and (14) yields

$$Q_3\left(t\right) = Q_0 - u_3\left[H_3\left(t\right) - H_3\left(0\right)\right] = Q_0 - u_3 H_0 h_p\left(t\right) \tag{15}$$

It is indicated from Eq. (15) that $Q_3\left(t\right)$ decreases with increasing $H_3\left(t\right)$, and that there is a linear relation between $Q_3\left(t\right)$ and $H_3\left(t\right)$.

Transforming Eqs. (1) and (2) into four ordinary differential ones using the method of characteristics, C^+ equations (the positive direction of the x-axis is A pointed to B) are obtained:

$$\begin{cases} \dfrac{g}{a_2}\dfrac{\mathrm{d}H}{\mathrm{d}t}+\dfrac{\mathrm{d}V}{\mathrm{d}t}=0 \\ \dfrac{\mathrm{d}x}{\mathrm{d}t}=a_2 \end{cases} \tag{16}$$

Integrating Eq. (16) along a C^+ line from A to P (Fig. 7) yields

$$\left(H_0 h_\mathrm{p}-\frac{1}{u_2}Q_2\right)\bigg|_{T_\mathrm{s}+\theta/2}=H_A(0)(\xi-0)\big|_{T_\mathrm{s}} \tag{17}$$

where $u_2=gf_2/a_2$.

Since the penstock is usually short (<700 m), the water head loss in the penstock, $H_\mathrm{wm}(0)$, is small compared with the pressure head, and can be neglected. In this study, $H_A(0)$ was approximately equal to H_0 , which was expressed as

$$H_A(0)=H_2(0)-H_\mathrm{wm}(0)=H_0-H_\mathrm{wm}(0)\approx H_0$$

$Q_2(T_\mathrm{s}+\theta/2)$ can be obtained by transforming Eq. (17):

$$Q_2(T_\mathrm{s}+\theta/2)=u_2H_0\left(h_\mathrm{p}^*-\xi^*\right) \tag{18}$$

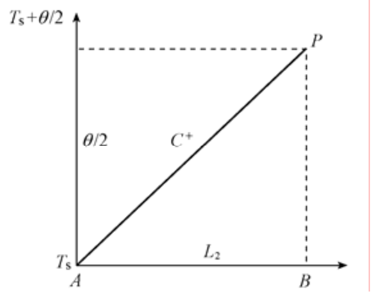

Fig. 7 Characteristic line of penstock

Combining Eqs. (6), (15), and (18) yields

$$Q_1\big|_{T_\mathrm{s}+L_2/a}=(Q_3-Q_2)\big|_{T_\mathrm{s}+L_2/a}=Q_0-u_3H_0h_\mathrm{p}^*+u_2H_0\left(\xi^*-h_\mathrm{p}^*\right) \tag{19}$$

The front of pressure wave ϕ_2 arrives at B at $t=\theta/2$. Meanwhile, $Q_1(t)$ and z begin to increase from 0. According to the continuity equation for the surge chamber,

$$z\big|_{T_\mathrm{s}+L_2/a_2}=\frac{1}{A_\mathrm{s}}\int_{\theta/2}^{T_\mathrm{s}+\theta/2}Q_1\mathrm{d}t=\frac{1}{A_\mathrm{s}}\int_{\theta/2}^{T_\mathrm{s}+\theta/2}(Q_3-Q_2)\mathrm{d}t \tag{20}$$

Since $Q_1(t)$, $Q_2(t)$, and $Q_3(t)$ are undetermined functions, their integrals, $z(T_\mathrm{s}+\theta/2)$, are undetermined. In our study, the valve was assumed to close linearly. In this condition $Q_1(t)$ can be viewed approximately as a linear function of t for $\theta\le t\le(T_\mathrm{s}+\theta/2)$. Therefore, $z(T_\mathrm{s}+\theta/2)$ can be approximated by

$$z\big|_{T_\mathrm{s}+\theta/2}=\frac{1}{A_\mathrm{s}}\int_{\theta/2}^{T_\mathrm{s}+\theta/2}Q_1\mathrm{d}t\approx\frac{1}{2}\left[Q_1(\theta/2)+Q_1(T_\mathrm{s}+\theta/2)\right]T_\mathrm{s}=\frac{1}{2}Q_1(T_\mathrm{s}+\theta/2)T_\mathrm{s} \tag{21}$$

Eq. (21) expresses $z(T_\mathrm{s}+\theta/2)$ as a function of $Q_1(T_\mathrm{s}+\theta/2)$, which can be determined by ξ^* and h_p^*.

Combining Eqs. (19) and (21) yields

$$z\big|_{T_\mathrm{s}+L_2/a}=\frac{T_\mathrm{s}}{2A_\mathrm{s}}Q_0-\frac{T_\mathrm{s}u_3H_0}{2A_\mathrm{s}}h_\mathrm{p}^*+\frac{T_\mathrm{s}u_2H_0}{2A_\mathrm{s}}\left(\xi^*-h_\mathrm{p}^*\right) \tag{22}$$

Substituting Eqs. (19) and (22) into Eq. (5), the following equation is obtained:

$$
\begin{aligned}
&H_0 h_{\mathrm{p}}^* + \frac{T_{\mathrm{s}} u_3 H_0}{2A_{\mathrm{s}}} h_{\mathrm{p}}^* - \frac{T_{\mathrm{s}}}{2A_{\mathrm{s}}} Q_0 - \frac{T_{\mathrm{s}} u_2 H_0}{2A_{\mathrm{s}}} \left(\xi^* - h_{\mathrm{p}}^*\right) \\
&= k\left[Q_0 - u_3 H_0 h_{\mathrm{p}}^* + u_2 H_0 \left(\xi^* - h_{\mathrm{p}}^*\right)\right]^2
\end{aligned} \tag{23}
$$

2.3 Interlocking equations of water hammer in penstock

2.3.1 Improved interlocking equations for conduit with surge chamber

In previous studies, the pressure head $H_2(t)$ at the upstream boundary was assumed to be constant in the course of deriving interlocking equations of water hammer. This is reasonable for a simple surge chamber located upstream with an infinitely large area. However, such interlocking equations are not applicable to hydraulic pipe problems with a throttled surge chamber, as $H_2(t)$ varies significantly with time upon closure of the valve. Modified interlocking equations should be derived for pressurized conduits with a throttled surge chamber located upstream.

Eq. (3) at the upstream end of the penstock (point B_2) can be written as

$$H_2(t) - H_2(0) = \phi_2(t - L_2/a_2) + \psi_2(t + L_2/a_2)$$

which, using $h_{\mathrm{p}}(t) = \left[H_2(t) - H_2(0)\right]/H_2(0)$, $H_2(0) = H_0$, and $L_2/a_2 = \theta/2$, can be transformed into

$$\phi_2(t - \theta/2) = -\psi_2(t + \theta/2) + H_0 h_{\mathrm{p}}(t) \tag{24}$$

Substituting $t = t_i - \theta/2$ into Eq. (24), the following equation is obtained:

$$\phi_2(t - \theta/2) = \phi_2(t_i - \theta/2 - \theta/2) = \phi_2(t_i - \theta) = -\psi_2(t_i) + H_0 h_{\mathrm{p}}(t_i - \theta/2)$$

which can be written in the general form

$$\psi_2(t) = -\phi_2(t - \theta) + H_0 h_{\mathrm{p}}(t - \theta/2) \tag{25}$$

or in the form

$$\psi_2^i = -\phi_2^{i-1} + H_0 h_{\mathrm{p}}^{i-1/2} \tag{26}$$

The superscript i denotes the value of the function at time $i\theta$, e.g., $\psi_2^i = \psi_2(i\theta)$. At the downstream end of the penstock (point A), Eqs. (3) and (4) can be written as

$$H_A^i = H_A^0 + \phi_2^i + \psi_2^i \tag{27}$$

$$V_A^i = V_A^0 - g/a_2\left(\phi_2^i - \psi_2^i\right) \tag{28}$$

where V_A and H_A are the flow velocity and pressure head at the downstream end of the penstock, respectively.

Substituting Eq. (26) into Eqs (27) and (28) yields

$$H_A^i = H_A^0 + \phi_2^i - \phi_2^{i-1} + H_0 h_{\mathrm{p}}^{i-1+1/2} \tag{29}$$

$$V_A^i = V_A^0 - g/a_2\left(\phi_2^i + \phi_2^{i-1} - H_0 h_{\mathrm{p}}^{i-1+1/2}\right) \tag{30}$$

After manipulations, the following equations can be obtained:

$$H_A^{i-1} + H_A^i = 2H_A^0 + \phi_2^i - \phi_2^{i-2} + H_0\left(h_{\mathrm{p}}^{i-1+1/2} + h_{\mathrm{p}}^{i-2+1/2}\right) \tag{31}$$

$$V_A^i - V_A^{i-1} = -g/a_2\left(\phi_2^i - \phi_2^{i-2}\right) + gH_0/a_2\left(h_{\mathrm{p}}^{i-1+1/2} - h_{\mathrm{p}}^{i-2+1/2}\right) \tag{32}$$

By eliminating the term $\left(\phi_2^i - \phi_2^{i-2}\right)$ from Eqs. (31) and (32), the following equation is obtained:

$$\begin{cases} H_A^1 - H_A^0 = -a_2/g\left(V_A^1 - V_A^0\right) + 2H_0 h_{\mathrm{p}}^{0+1/2} \\ H_A^1 + H_A^2 - 2H_A^0 = -a_2\big/g\left(V_A^2 - V_A^1\right) + 2H_0 h_{\mathrm{p}}^{1+1/2} \\ \qquad\qquad\vdots \\ H_A^{i-1} + H_A^i - 2H_A^0 = -a_2\big/g\left(V_A^i - V_A^{i-1}\right) + 2H_0 h_{\mathrm{p}}^{i-1+1/2} \end{cases} \tag{33}$$

When pressure head at the entrance to the surge chamber just below the orifice is constant, i.e., $h_{\mathrm{p}}(t) \equiv 0$, Eq. (33) becomes identical to the interlocking equations derived by Allievi (1913). It can be seen that Allievi's interlocking equations are a special case of Eq. (33).

To generalize, the improved interlocking equations may be expressed in dimensionless terms by defining the following quantities: $\xi^i = \left(H_A^i - H_A^0\right)\big/H_0$, $V_A^i\big/V_0 = v^i$, and $\mu = a_2 V_0\big/\left(2gH_0\right)$. Substituting these terms into Eq. (33) yields the following equation:

$$\begin{cases} \xi^1 = -2\mu\left(v^1 - v^0\right) + 2h_{\mathrm{p}}^{0+1/2} \\ \xi^1 + \xi^2 = -2\mu\left(v^2 - v^1\right) + 2h_{\mathrm{p}}^{1+1/2} \\ \qquad\qquad\vdots \\ \xi^{i-1} + \xi^i = -2\mu\left(v^i - v^{i-1}\right) + 2h_{\mathrm{p}}^{i-1+1/2} \end{cases} \tag{34}$$

2.3.2 Interlocking equations in condition of final water hammer

For the system with a valve located at the downstream end of the penstock, the Bernoulli equation is assumed to be valid (Wylie and Streeter 1993):

$$v(t) = \tau(t)\sqrt{1 + \xi(t)} \tag{35}$$

where $v(t) = V_A(t)\big/V_A(0)$, and $\tau(t)$ is the relative valve opening.

Substituting this into Eq. (34) yields the following equation:

$$\mu\left(\tau^n\sqrt{1+\xi^n} - \tau^{n-1}\sqrt{1+\xi^{n-1}}\right) = -\frac{\xi^n + \xi^{n-1}}{2} + h_{\mathrm{p}}^{n-1/2} \tag{36}$$

The hydraulic system mentioned before (Figs. 4 and 5), in which $\theta = 1.4$ s and $T_{\mathrm{s}} = 13.0$ s, is considered, and some distinguishing features of the functions h_{p} and ξ, which will be used for deriving equations in the next section, are found:

(1) The curve of $\xi(t)$ turns at time 0, θ, T_{s}, and $T_{\mathrm{s}} + n\theta$ (n = 1, 2, 3…), and the curve of $h_{\mathrm{p}}(t)$ turns at time $T_{\mathrm{s}} + \left(n + 1/2\right)\theta$, ($n$ = 1, 2, 3…).

(2) Provided that closure of the valve is continuous and differentiable with respect to time, $\xi(t)$ and $h_{\mathrm{p}}(t)$ are continuous and differentiable with respect to time in any time intervals between any two successive points mentioned above.

Since ξ is continuous and differentiable in the time interval $t \in \left[\theta, T_{\mathrm{s}}\right]$, employing the Taylor expansions of $\tau^n\sqrt{1+\xi^n}$ and $\tau^{n-1}\sqrt{1+\xi^{n-1}}$ to the second order at time instant $\left(n - 1/2\right)\theta$ yields the following equations:

$$\tau^n\sqrt{1+\xi^n}=\tau\sqrt{1+\xi}\Big|_{n-1/2}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\tau\sqrt{1+\xi}\right)\left(\frac{\theta}{2}\right)\Big|_{n-1/2}+\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\tau\sqrt{1+\xi}\right)\left(\frac{\theta}{2}\right)^2\Big|_{n-1/2}+$$
$$\frac{1}{6}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\left(\frac{\theta}{2}\right)^3\Big|_{\alpha}$$
$$\tau^{n-1}\sqrt{1+\xi^{n-1}}=\tau\sqrt{1+\xi}\Big|_{n-1/2}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\tau\sqrt{1+\xi}\right)\left(-\frac{\theta}{2}\right)\Big|_{n-1/2}+\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\tau\sqrt{1+\xi}\right)\left(-\frac{\theta}{2}\right)^2\Big|_{n-1/2}+$$
$$\frac{1}{6}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\left(-\frac{\theta}{2}\right)^3\Big|_{\beta}$$
where $\alpha\in[n-1,n-1/2]$, and $\beta\in[n-1/2,n]$.

Subtraction of the two equations above yieds the following equation:

$$\tau^n\sqrt{1+\xi^n}-\tau^{n-1}\sqrt{1+\xi^{n-1}}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\tau\sqrt{1+\xi}\right)\theta\Big|_{n-1/2}+\frac{1}{6}\left[\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\alpha}+\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\beta}\right]\left(\frac{\theta}{2}\right)^3$$
$$=\theta\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\sqrt{1+\xi}+\tau\frac{1}{2\sqrt{1+\xi}}\frac{\mathrm{d}\xi}{\mathrm{d}t}\right)\Big|_{n-1/2}+\frac{1}{6}\left[\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\alpha}+\frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\beta}\right]\left(\frac{\theta}{2}\right)^3 \tag{37}$$

At $t\approx T_{\mathrm{s}}$ and $\tau\approx 0$, Allievi (1913) found that final water hammer occurs for $\tau_0\mu>1$. This indicates that the maximum water hammer pressure occurs at T_{s}:

$$\tau\frac{1}{2\sqrt{1+\xi}}\frac{\mathrm{d}\xi}{\mathrm{d}t}\approx 0$$

For linear valve closure,

$$\frac{\mathrm{d}\tau}{\mathrm{d}t}=-\frac{\tau_0}{T_{\mathrm{s}}}$$

Magnitude analysis of the right-side term in Eq. (37) yields the following equations:

$$\begin{cases} R_1=\theta\dfrac{\mathrm{d}\tau}{\mathrm{d}t}\sqrt{1+\xi}=O\left(\tau_0\dfrac{\theta}{T_{\mathrm{s}}}\right)O\left(\sqrt{1+\xi}\right) \\ R_2=\dfrac{1}{6}\left[\dfrac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\alpha}+\dfrac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tau\sqrt{1+\xi}\right)\Big|_{\beta}\right]\left(\dfrac{\theta}{2}\right)^3=O\left(\tau_0\dfrac{\theta^3}{T_{\mathrm{s}}^3}\right)O\left(\sqrt{1+\xi}\right) \end{cases} \tag{38}$$

In hydro-electric power plants, $\theta/T_{\mathrm{s}}\approx 1/10$. It can be seen that R_2 is two orders of magnitude smaller than R_1, and can be neglected. Therefore, Eq. (37) can be rewritten as

$$\tau_n\sqrt{1+\xi_n}-\tau_{n-1}\sqrt{1+\xi_{n-1}}\approx-\frac{\tau_0\theta}{T_{\mathrm{s}}}\sqrt{1+\xi_{n-1/2}} \tag{39}$$

Using the Taylor expansions of ξ_n and ξ_{n-1} at time $(n-1/2)\theta$ to the first order and then adding them to each other leads to the following equation:

$$\frac{\xi_n + \xi_{n-1}}{2\mu} = \frac{\xi_{n-1/2}}{\mu} + \frac{1}{4\mu}\left(\left.\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2}\right|_{\kappa} + \left.\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2}\right|_{\lambda} \right)\left(\frac{\theta}{2}\right)^2 \tag{40}$$

where $\kappa \in [n-1, n-1/2]$ and $\lambda \in [n-1/2, n]$. S_1 and S_2 are set as follows:

$$\begin{cases} S_1 = \dfrac{\xi_{n-1/2}}{\mu} = O\left(\dfrac{1}{\mu}\right) O(\xi) \\ S_2 = \dfrac{1}{4\mu}\left(\left.\dfrac{\mathrm{d}^2\xi}{\mathrm{d}t^2}\right|_{\kappa} + \left.\dfrac{\mathrm{d}^2\xi}{\mathrm{d}t^2}\right|_{\lambda} \right)\left(\dfrac{\theta}{2}\right)^2 = O\left(\dfrac{1}{\mu}\dfrac{\theta^2}{T_{\mathrm{s}}^2}\right) O(\xi) \end{cases} \tag{41}$$

S_2 is two orders of magnitude smaller than S_1, and can be neglected. Therefore, Eq. (40) can be rewritten as

$$\frac{\xi_n + \xi_{n-1}}{2\mu} \approx \frac{\xi_{n-1/2}}{\mu} \tag{42}$$

Substituting Eqs. (42) and (39) and introducing a dimensionless Allievi constant $\sigma = \dfrac{\tau_0 L_2 Q_0}{f_2 g H_0 T_{\mathrm{s}}}$ into Eq. (36) yields the following equation:

$$-\sigma\sqrt{1+\xi^{n-1/2}} = -\xi^{n-1/2} + h_{\mathrm{p}}^{n-1/2} \tag{43}$$

Eq. (43) is valid when $n\theta \approx T_{\mathrm{s}}$. It can be viewed as the fitting curve of ξ and h_{p} in the adjacent time interval of T_{s}. Eq. (43) is approximately valid at T_{s}, yielding

$$-\sigma\sqrt{1+\xi^*} = -\xi^* + h_{\mathrm{p}}(T_{\mathrm{s}}) \tag{44}$$

Eq. (44) indicates the relation between $h_{\mathrm{p}}(T_{\mathrm{s}})$ and ξ^*. Because $h_{\mathrm{p}}(t)$ is a smooth function in a time interval $0 \sim (T_{\mathrm{s}} + \theta/2)$, Taylor expansion of $h_{\mathrm{p}}(T_{\mathrm{s}})$ at time $T_{\mathrm{s}} + \theta/2$ to a first order term yields

$$h_{\mathrm{p}}(T_{\mathrm{s}}) = h_{\mathrm{p}}^* - \frac{\theta}{2}\left.\frac{\mathrm{d}h_{\mathrm{p}}}{\mathrm{d}t}\right|_{T_{\mathrm{s}}+\theta/2} + \left.\frac{1}{2}\frac{\mathrm{d}^2 h_{\mathrm{p}}}{\mathrm{d}t^2}\left(\frac{\theta}{2}\right)^2\right|_{\gamma} \tag{45}$$

where $\gamma \in [T_{\mathrm{s}}, T_{\mathrm{s}} + \theta/2]$. Magnitude analysis of the second and the third terms of Eq. (45) yields

$$\begin{cases} T_1 = \dfrac{\theta}{2}\dfrac{\mathrm{d}h_{\mathrm{p}}}{\mathrm{d}t} = O\left(\dfrac{\theta}{T_{\mathrm{s}}}\right) O\left(h_{\mathrm{p}}^*\right) \\ T_2 = \dfrac{\mathrm{d}^2 h_{\mathrm{p}}}{\mathrm{d}t^2}\theta^2 = O\left(\dfrac{\theta^2}{T_{\mathrm{s}}^2}\right) O\left(h_{\mathrm{p}}^*\right) \end{cases} \tag{46}$$

It can be clearly seen that T_2 is one order of magnitude smaller than T_1, and can be neglected. In $\theta/2 \sim (T_{\mathrm{s}} + \theta/2)$, as mentioned before, the flow rate into the surge chamber can be approximated by a linear function with time, viz. $Q_1(t) \propto (t - \theta/2)$, so we can obtain

$$z(t) = \int_{\theta/2}^{t} Q_1 \mathrm{d}t \propto (t-\theta/2)^2, \qquad kQ_1^2(t) \propto (t-\theta/2)^2 \tag{47}$$

$$H_0 h_{\mathrm{p}}(t) = z(t) + kQ_1^2(t) \propto (t-\theta/2)^2 \tag{48}$$

The derivative of parabola $y = x^2$ at $x = x_0$ is $2x_0$. The slope of the secant line of $y = x^2$ at $x = x_0$ is x_0. The derivative of $h_{\mathrm{p}}(t)$ is twice as large as the secant slope of $h_{\mathrm{p}}(t)$. Magnitude analysis of $h_{\mathrm{p}}(t)$ yields the following equations:

$$\left.\frac{\mathrm{d}h_{\mathrm{p}}}{\mathrm{d}t}\right|_{T_{\mathrm{s}}+\theta/2}=2\frac{h_{\mathrm{p}}^{*}}{T_{\mathrm{s}}} \tag{49}$$

$$h_{\mathrm{p}}(T_{\mathrm{s}})=h_{\mathrm{p}}^{*}-\theta\frac{h_{\mathrm{p}}^{*}}{T_{\mathrm{s}}} \tag{50}$$

We set $\varsigma=\theta/T_{\mathrm{s}}$, which is about 1/10 in practical hydraulic engineering projects. Combining Eqs. (44) and (50) yields the following equation:

$$-\sigma\sqrt{1+\xi^{*}}=-\xi^{*}+h_{\mathrm{p}}^{*}(1-\varsigma) \tag{51}$$

2.4 Analytical formulas of h_{p}^{*} and ξ^{*}

The two variables ξ^{*} and h_{p}^{*} can be solved from Eqs. (23) and (51). Eq. (51) can be rewritten as

$$h_{\mathrm{p}}^{*}=\frac{\xi^{*}-\sigma\sqrt{1+\xi^{*}}}{1-\varsigma} \tag{52}$$

Substituting Eq. (52) into Eq. (23) and manipulating the resultant equation yields a quartic equation of $\sqrt{1+\xi^{*}}$, the solution of which is complicated. An approximate analytical solution can be derived using the following expansion of $\sqrt{1+\xi^{*}}$ at $\xi^{*}=0$:

$$\sqrt{1+\xi^{*}}=1+\frac{1}{2}\xi^{*}+O\left[\left(\xi^{*}\right)^{2}\right] \tag{53}$$

Since ξ^{*} is much smaller than 1, the second order term can be ignored. Thus, Eq. (53) can be rewritten as

$$\sqrt{1+\xi^{*}}\approx 1+\frac{1}{2}\xi^{*} \tag{54}$$

Such an approximation was used by Alleivi (1913):

$$H_{0}h_{\mathrm{p}}^{*}=\left.kQ_{1}^{2}\right|_{T_{\mathrm{s}}+\theta/2}+\left.z\right|_{T_{\mathrm{s}}+\theta/2} \tag{55}$$

Eq. (55) shows that h_{p}^{*} may be viewed as the sum of pressure difference and the increase of water level in the surge chamber. If the orifice is small enough, $z(T_{\mathrm{s}}+\theta/2)$ is also small compared with $H_{0}h_{\mathrm{p}}^{*}$, and $\left.Q_{1}\right|_{T_{\mathrm{s}}+\theta/2}$ can be estimated as Q_{0}. Therefore, h_{p}^{*} can be estimated by the following equation:

$$h_{\mathrm{p}}^{*}\approx kQ_{0}^{2}/H_{0} \tag{56}$$

It is shown from Eq. (56) that there is an approximately linear proportion between h_{p}^{*} and k, provided that Q_{0} remains stable. Combining Eqs. (51) and (54) yields a linear function of ξ^{*} and h_{p}^{*}, and substituting it into Eq. (23) leads to a quadratic function of h_{p}^{*}:

$$\begin{aligned}&H_{0}h_{\mathrm{p}}^{*}+\frac{T_{\mathrm{s}}}{2A_{\mathrm{s}}}H_{0}h_{\mathrm{p}}^{*}\left(u_{3}+\frac{\sigma-2\varsigma}{2-\sigma}u_{2}\right)-\frac{T_{\mathrm{s}}}{2A_{\mathrm{s}}}\left(Q_{0}+\frac{2\sigma}{2-\sigma}u_{2}H_{0}\right)\\&=k\left[Q_{0}+\frac{2\sigma}{2-\sigma}u_{2}H_{0}-\left(\frac{\sigma-2\varsigma}{2-\sigma}u_{2}+u_{3}\right)H_{0}h_{\mathrm{p}}^{*}\right]^{2}\end{aligned} \tag{57}$$

Dimensionless terms are defined as follows:

$$\lambda=\frac{T_{\mathrm{s}}Q_0}{2A_{\mathrm{s}}H_0},\quad \pi=1+\frac{2\sigma}{(2-\sigma)}\frac{u_2H_0}{Q_0},\quad \nu=\left(u_3+\frac{\sigma-2\varsigma}{2-\sigma}u_2\right)\frac{H_0}{Q_0},\quad \eta=\frac{kQ_0^2}{H_0}$$

Substituting them into Eq. (57), the following equation in dimensionless form is obtained:

$$h_{\mathrm{p}}^*+\lambda\nu h_{\mathrm{p}}^*-\lambda\pi=\eta\left(\pi-\nu h_{\mathrm{p}}^*\right)^2 \tag{58}$$

The reasonable solution of Eq. (58) is

$$h_{\mathrm{p}}^*=\frac{c_2-\sqrt{c_2^2-4c_1c_3}}{2c_1} \tag{59}$$

in which $c_1=\eta\nu^2$, $c_2=2\eta\pi\nu+\lambda\nu+1$, and $c_3=\eta\pi^2+\lambda\pi$. Setting $c_4=h_{\mathrm{p}}^*\left(1-\varsigma\right)+1$, and substituting Eq. (59) into Eq. (51), the following equation is obtained:

$$\xi^*=\frac{1}{2}\left(\sigma^2+2c_4+\sqrt{\sigma^2+4c_4}\right)-1 \tag{60}$$

Eqs. (59) and (60) are the analytical formulas of h_{p}^* and ξ^*, which are derived from the classic water hammer equations using the approximate method dealing with the term $\sqrt{1+\xi^*}$.

3 Results and discussion

The conditions for using Eqs. (59) and (60) are (1) that the upstream tunnel is long enough that at time $T_{\mathrm{s}}+\theta/2$ the front of ψ_3 has not yet arrived at the junction, i.e., $L_3\geq a_3T_{\mathrm{s}}$; (2) that the type of water hammer is final water hammer, i.e., $\tau_0\mu>1$, and T_{s} is much larger than θ (the order of magnitude of T_{s} is close to that of 10θ) to ensure that indirect water hammer occurs; and (3) that the opening of the valve/turbine decreases linearly. In available experiments, the fast valve and needle valve are adopted, neither of which closes linearly. Also, in many experiments, $T_{\mathrm{s}}<\theta$, and direct water hammer is generated.

Zienkiewicz and Hawkins (1954) used the Schnyder-Bergeron graphical method to calculate transmission pressure, achieving good agreement between theoretical and experimental results. The graphical method has been replaced by numerical methods that have better accuracy and efficiency, so the results have also verified the numerical methods. Peng and Yang (1986) computed the transient pressure and found that the numerical result was in good agreement with the experiment. This verified that the basic assumptions are reasonable. Prenner and Drobir (1997) conducted an experiment using four different throttle-type orifices to study the pressure wave transmission through the surge chamber, and also made a numerical calculation using the method of characteristics (MOC), which showed good agreement with the experiment. All of these studies show that the MOC leads to good agreement with the experimental data. In this study, results obtained using the described analytical formula were compared with the numerical results obtained using the MOC to examine the validity of the approximate equation.

Numerical solution of one-dimensional fluid transient flow in pipe systems has been developed for half a century. The MOC, which has desirable accuracy, simplicity, and numerical efficiency, is very popular. The characteristics method of one-dimensional fluid transient flow and the boundary treatment technique can be found in some standard reference books (Wylie and Streeter. 1993; Chaudhry 1987). The time step in this study was 10^{-2} s. The results tend to be convergent as the number of grid cells increases. A relative numerical error of less than 10^{-6} was adopted. The downstream end condition was treated as a valve. The formula of the head loss coefficient used here was Eq. (9). In this study, the contraction coefficient C = 0.700. We set $w = f_0 / f_3$. Other parameters were held constant; h_p^* and ξ^* increased with decreasing w. The validity of the present analytical formulas for a hydraulic piping system with a surge chamber was examined across a range from 0.1 to 1.0, which covers the scope of practical situations. The physical and geometric parameters of the system were as follows: the area of the surge chamber was 450.0 m^2, and the area of the orifice was in the range of 11.309 7 m^2 to 113.097 3 m^2; the turbine was simplified as a valve and its closure time was 10 s; the water levels of the upstream reservoir and downstream river were 1 658.0 m and 1 314.6 m, respectively; and other parameters associated with a penstock and a tunnel are given in Table 1.

Table 1 Geometric parameters of tunnel and penstock

System	Length (m)	Diameter (m)	Roughness	Area (m^2)
Penstock	700.0	10.0	0.013	78.539 8
Tunnel	17 000.0	12.0	0.014	113.097 3

Analytical and numerical results of $H_0 h_p^*$ and $H_0 \xi^*$ are given in Table 2. It can be seen from Table 2 that analytical results of $H_0 h_p^*$ are in good agreement with the numerical ones. The maximum relative error between them of the ten cases where w ranges from 0.1 to 1.0 was less than 0.5%. The maximum relative error of $H_0 \xi^*$ of the ten cases was around 3%.

Table 2 Analytical and numerical results of $H_0 h_p^*$ and $H_0 \xi^*$

w	$H_0 h_p^*$			$H_0 \xi^*$		
	Theoretical value (m)	Numerical result (m)	Relative error (%)	Theoretical value (m)	Numerical result (m)	Relative error (%)
1/10	109.95	110.43	0.44	137.61	140.05	1.74
2/10	43.33	43.48	0.33	75.17	76.68	1.98
3/10	24.15	24.22	0.30	57.14	58.52	2.36
4/10	16.64	16.68	0.27	50.07	51.41	2.61
5/10	13.03	13.07	0.25	46.68	48.00	2.76
6/10	11.06	11.09	0.23	44.82	46.14	2.86
7/10	9.88	9.90	0.22	43.71	45.02	2.92
8/10	9.12	9.14	0.21	42.99	44.31	2.96
9/10	8.61	8.63	0.20	42.51	43.82	2.99
10/10	8.25	8.26	0.19	42.17	43.48	3.01

The derivations of h_p^* and ξ^* are based on Eqs. (1) and (2), which are derived by neglecting conduit friction, so the formulas h_p^* and ξ^* did not take friction into account. In order to improve the accuracy, friction was introduced into the formula.

At time $t = 0$, the flow rate in the penstock is Q_0 and the head loss due to friction in the penstock is $H_{\mathrm{wm}}(0)$. At time $t = T_{\mathrm{s}} + \theta/2$, the flow velocity in the penstock is reduced to nearly zero, so the friction is reduced to nearly zero. The decrease of flow velocity in the tunnel is relatively small. Head recovery due to friction decrease can be estimated by $H_{\mathrm{wm}}(0)/H_0$ for ξ^*, and is negligible for h_{p}^*. The modified formula for ξ^* is

$$\xi^* = \frac{1}{2}\left(\sigma^2 + 2c_4 + \sqrt{\sigma^2 + 4c_4}\right) - 1 + H_{\mathrm{wm}}(0)/H_0 \tag{61}$$

Table 3 shows that the modified formula of ξ^*, Eq. (61), is in better agreement with the numerical results than Eq. (60). The relative error increases with the decrease of *w*. This is due to the fact that the value of ξ^* increases with the decrease of *w*. The residual error of approximate treatment of Eq. (43) also increases. The maximum relative error of all ten cases was less than 1%.

Table 3 Theoretical value of $H_0\xi^*$ calculated by Eq. (61) and its relative error

w	$H_0\xi^*$ (m)	Relative error (%)	w	$H_0\xi^*$ (m)	Relative error (%)
1/10	138.83	0.87	6/10	46.05	0.21
2/10	76.39	0.38	7/10	44.93	0.20
3/10	58.36	0.27	8/10	44.22	0.20
4/10	51.29	0.23	9/10	43.73	0.20
5/10	47.90	0.21	10/10	43.39	0.20

In this section, ξ^* is modified by adding a penstock friction term. Analytical results of ξ^* and h_{p}^* are compared with numerical results obtained using the MOC, showing that they are in a good agreement.

4 Conclusions

In this study, water hammer and transmitted pressure in a hydro-electric power plant with a long diversion tunnel and a throttled surge chamber were examined. Two equations, namely a tunnel-surge chamber-penstock joint equation and a water hammer interlocking equation for a penstock with a surge chamber located upstream were derived, in which the maximum water hammer pressure ξ^* at the valve and the maximum transmitted pressure h_{p}^* were two unknown variables. The analytical formulas of h_{p}^* and ξ^* were deduced by solving these two equations. Taking friction in the penstock into account, the analytical formula of ξ^* was improved in accuracy.

The results obtained using the proposed analytical formulas are in good agreement with the numerical results obtained using the method of characteristics for various sizes of the orifice.

Under the assumption that the reflected wave from the inlet of the division tunnel does not arrive at the bottom of the surge tank at time $T_{\mathrm{s}} + \theta/2$, the proposed formulas are valid for hydro-electric power plants with long diversion tunnels. For short tunnels, the reflected wave from the inlet of the tunnel has to be taken into consideration. For such a case, an analytical study will be quite complicated, but deserves further exploration.

In this study, h_{p}^* and ξ^* were examined under the conditions of linear valve closure.

Water hammer pressure and transmitted pressure for nonlinear valve closure will be the subject of a future paper.

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