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Calculation of passive earth pressure of cohesive soil based on Culmann's method

Hai-feng LU*, Bao-yuan YUAN

College of Earth Science and Engineering, Hohai University, Nanjing 210098, P. R. China

Abstract: Based on the sliding plane hypothesis of Coulumb earth pressure theory, a new method for calculation of the passive earth pressure of cohesive soil was constructed with Culmann's graphical construction. The influences of the cohesive force, adhesive force, and the fill surface form were considered in this method. In order to obtain the passive earth pressure and sliding plane angle, a program based on the sliding surface assumption was developed with the VB.NET programming language. The calculated results from this method were basically the same as those from the Rankine theory and Coulumb theory formulas. This method is conceptually clear, and the corresponding formulas given in this paper are simple and convenient for application when the fill surface form is complex.

Key words: *Coluomb earth pressure theory; Culmann's graphical construction; retaining wall; passive earth pressure; cohesive soil*

1 Introduction

Retaining walls are widely used in industrial and civil construction, road transport, water conservancy and hydropower, port construction, and other projects. In view of safety and economy, the strength and distribution of the earth pressure on the retaining wall, and its factors, must be comprehensively considered in design.

Rankine theory, Coulumb theory, some graphical construction methods based on them, and some numerical methods are commonly used to calculate the passive earth pressure. Hu and Tan (2009) proposed a formula for passive earth pressure on cohesive soil. Fang et al. (2002) estimated the passive earth pressure by introducing the critical state concept to either Terzaghi or Coulomb theory. Using variational calculus and Lagrange multipliers, Li and Liu (2007) proposed a calculation method for the passive earth pressure on a retaining wall. Clough and Woodward (1967), Clough and Duncan (1971), and Wang (2000) used finite elements to analyze the retaining wall behavior. Cheng (2003) solved the slip line equations by rotation of axes in order to determine the lateral earth pressure with the presence of seismic loading under general conditions. Sobrou and Macuh (2002) calculated the passive earth pressure coefficients of an inclined wall and a sloping backfill in the general case

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*Corresponding author (e-mail: *luhaifeng7571@126.com*)
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based on rotational log-spiral failure mechanisms of the upper-bound theorem of limit analysis. Chen and Li (1998) used the generalized method of slices to determine the earth pressures and applied it to various types of supports. Wang and Que (2003) improved Culmann's graphical construction for calculating the active earth pressure by combining geometrical and physical methods. Zhu and Qian (2000) proposed a new procedure to determine the passive earth pressure coefficient using triangular slices according to the limit equilibrium method. Vrecl-Kojc and Skrabl (2007) presented a modified three-dimensional failure mechanism for determining the three-dimensional passive earth pressure coefficient using the upper bound theorem based on the limit analysis theory. These solutions provided effective ways for calculating the passive earth pressure. However, further work is required to increase their effectiveness.

We know that Rankine theory requires a smooth vertical interface between the retaining wall and horizontal fill surface. Thus, it cannot easily be applied in practical engineering. Coulumb earth pressure theory is also difficult to apply in practical engineering because it is based on the hypothesis of non-cohesive backfill. The numerical methods are too theoretical for engineers and technicians to master. Culmann's graphical construction, based on Coulumb earth pressure theory, was used to calculate earth pressure of non-cohesive fill according to the wedge theory. It can be used in the conditions of an irregular fill surface or a fill surface with loads, and is preferable because of these advantages. However, few articles can be found on the passive earth pressure calculation for cohesive fill with Culmann's graphical construction.

Based on the sliding plane hypothesis of Coulumb earth pressure theory, this paper proposes a new method for calculating the passive earth pressure using Culmann's graphical construction, with consideration of factors such as the size and obliquity of the back of the retaining wall, the cohesive force on the sliding surface, the adhesive force on the interface of the retaining wall, the irregular fill surface, and the influence of the fill surface with loads.

2 Basic principles

2.1 Calculation of passive earth pressure of non-cohesive soil

The principle of calculation of the passive earth pressure of non-cohesive soil with Culmann's method (Gu 2002) is described here, with a retaining wall of unit length (1 m) as an example (Fig. 1). Parameters of the wall and backfill include the following: α is the obliquity of the back of the retaining wall, β is the fill surface obliquity, H is the height of the retaining wall, ϕ is the internal friction angle of the fill, δ is the external friction angle, and q is the load on the fill surface. A stochastic sliding wedge ABC is shown in Fig. 1, where θ is the sliding surface obliquity, $\boldsymbol{G}$ is the gravity acting on the sliding wedge, $\boldsymbol{R}$ is the soil reaction, and $\boldsymbol{E}$ is the retaining wall reaction. Eq. (1) and Eq. (2) are obtained according to the law of sines (Gu 2002).

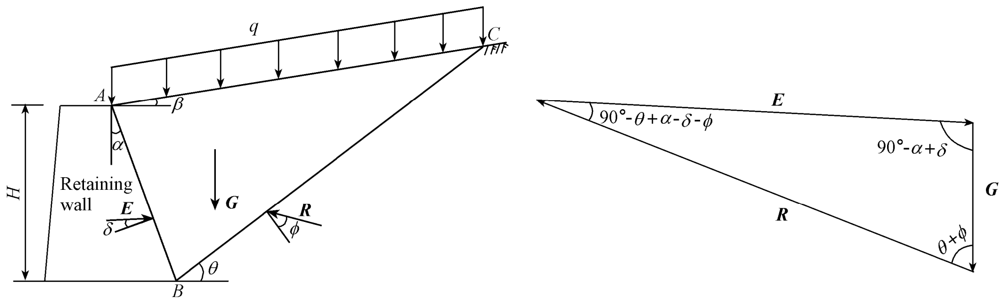


Fig. 1 Force diagram for passive earth pressure calculation of non-cohesive soil sliding wedge

$$\frac{|\boldsymbol{E}|}{\sin(\theta+\phi)}=\frac{|\boldsymbol{G}|}{\sin(90^\circ-\theta+\alpha-\delta-\phi)} \tag{1}$$

$$|\boldsymbol{E}|=|\boldsymbol{G}|\frac{\sin(\theta+\phi)}{\sin(90^\circ-\theta+\alpha-\delta-\phi)} \tag{2}$$

The gravity acting on the sliding wedge can be described as (Gu 2002)

$$|\boldsymbol{G}|=\rho g \mathrm{S}_{\Delta ABC}=\frac{1}{2}\rho g l_{AB} l_{BC}\sin(90^\circ-\theta+\alpha) \tag{3}$$

Where ρ is the fill density, g is the gravitational acceleration, and l_{AB} and l_{BC} are the lengths of *AB* and *BC*, respectively. Then,

$$|\boldsymbol{G}|=\frac{1}{2}\rho g H^2\frac{\cos(\alpha-\beta)\cos(\theta-\alpha)}{\cos^2\alpha\sin(\theta-\beta)} \tag{4}$$

If there is a load q on the sliding wedge of the fill surface, the vertical downward force is $\boldsymbol{G}_{\mathrm{T}}=\boldsymbol{G}+\boldsymbol{G}'$, and

$$|\boldsymbol{G}'|=qH\frac{\cos(\theta-\alpha)\cos\beta}{\cos\alpha\sin(\theta-\beta)} \tag{5}$$

Eq. (6) is obtained according to Eqs. (2) through (5):

$$|\boldsymbol{E}|=\left[\frac{1}{2}\rho g H^2\frac{\cos(\alpha-\beta)\cos(\theta-\alpha)}{\cos^2\alpha\sin(\theta-\beta)}+qH\frac{\cos(\theta-\alpha)\cos\beta}{\cos\alpha\sin(\theta-\beta)}\right]\frac{\sin(\theta+\phi)}{\sin(90^\circ-\theta+\alpha-\delta-\phi)} \tag{6}$$

BC is taken as a random sliding surface. In order to calculate the Coulumb passive earth pressure $\boldsymbol{E}_{\mathrm{P}}$, the extremum of Eq. (6) must be obtained. Assuming that $\mathrm{d}|\boldsymbol{E}|/\mathrm{d}\theta=0$, the angle of rupture θ_0 can be obtained. The formula of $\boldsymbol{E}_{\mathrm{P}}$ can be formed from Eq. (6) when θ is θ_0.

The passive earth pressure is calculated according to the force balance principle in Culmann's graphical construction. As shown in Fig. 2(a), two straight lines *W* and *L* are made from the heel point *B*. The angle formed by line *W* and the horizon is ϕ, and the angle formed by line *L* and line *W* is $90^\circ-\alpha+\delta$. *BD* is intercepted along line *W* at a certain force scale, so that the magnitude of the gravity **G** of the sliding wedge *ABC* can be expressed with the length of *BD*, i.e., $l_{BD}=|\boldsymbol{G}|$. From the point *D*, line *DF*, which is parallel to line *L* and has an intersect point *F* with the sliding plane *BC*, is made, and ΔBDF is formed, as shown in Fig. 2(a).

Comparing Fig. 1(b) with Fig. 2(a), it is easy to see that side DF in ΔBDF corresponds to the retaining wall reaction $\boldsymbol{E}$**,** and the magnitude of $\boldsymbol{E}$ can be expressed with the length of DF, i.e., $l_{DF} = |\boldsymbol{E}|$. Thus, BF is equivalent to the soil reaction $\boldsymbol{R}$ of the corresponding sliding plane BC, i.e., $l_{BF} = |\boldsymbol{R}|$. If a series of sliding surfaces, BC_1, BC_2, etc., are assumed, the method described above can be used to obtain the retaining wall reaction corresponding to each sliding surface. The minimum retaining wall reaction is the passive earth pressure ($\boldsymbol{E}_{\mathrm{p}}$) of the retaining wall, which corresponds to DF in Fig. 2(b). This method has a significant advantage: it can obtain the passive earth pressure in a variety of fill surfaces.

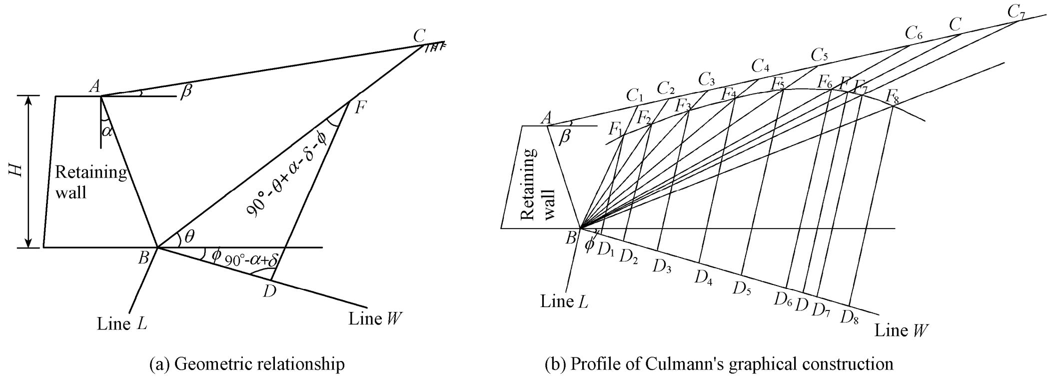


Fig. 2 Culmann's graphical construction to determine passive earth pressure

When Culmann's method is used to calculate the active earth pressure, line W, which has an angle ϕ with the horizontal plane, should be painted over the horizon. Other constructive steps are the same as those in the method for passive earth pressure calculation (Gu 2002). The maximum value of the retaining wall reaction ($\boldsymbol{E}_{\mathrm{a}}$) is the active earth pressure (Fig. 3).

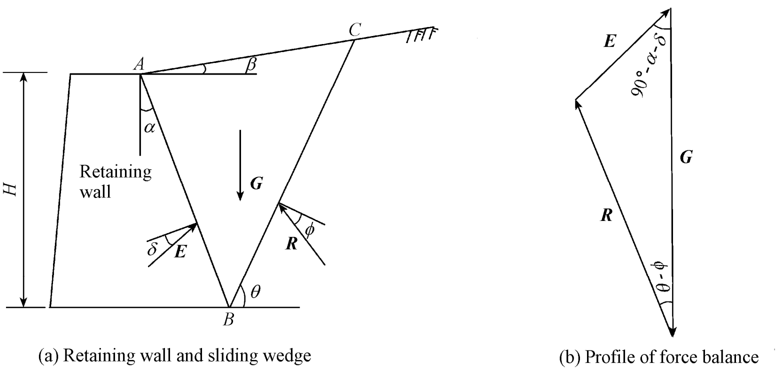


Fig. 3 Force diagram for active earth pressure calculation of non-cohesive soil sliding wedge

2.2 Calculation of passive earth pressure of cohesive soil

Culmann's graphical construction is based on the sliding plane hypothesis of Coulumb earth pressure theory. In this study, the earth pressure of cohesive soil was calculated according to Culmann's graphical construction under this assumption.

In accordance with the above principle, the passive earth pressure calculation method, which takes the cohesive force c_s and the adhesive force c_w into consideration, can be obtained. A stochastic sliding wedge ABC is shown in Fig. 4. Vectors $\overrightarrow{MQ}$ and $\overrightarrow{IK}$ represent the retaining wall reaction $\boldsymbol{E}$ and soil reaction $\boldsymbol{R}$, respectively.

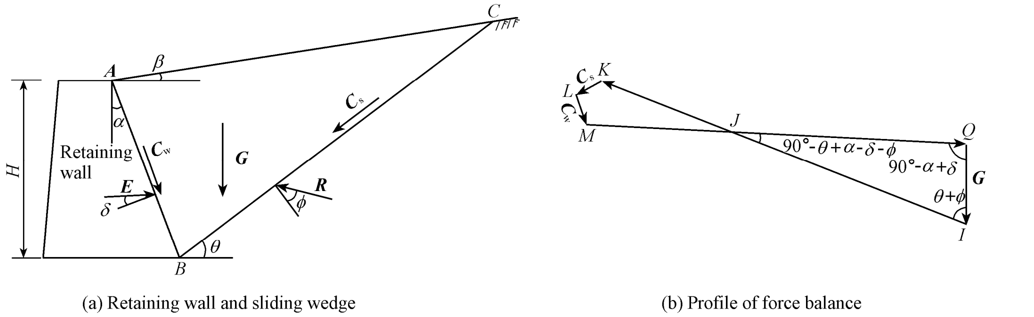


Fig. 4 Force diagram for passive earth pressure calculation of cohesive soil sliding wedge

When Fig. 4(b) is compared to Fig. 1(b), it is seen that the force diagram changes when considering the influence of c_s and c_w. $\overrightarrow{JQ}$ (Fig. 4(b)) is the retaining wall reaction when c_s and c_w are not taken into consideration. The resultant cohesive force on the sliding surface is $|\boldsymbol{C}_s| = c_s l_{BC}$ and the resultant adhesive force on the interface of the retaining wall is $|\boldsymbol{C}_w| = c_w l_{AB}$. Considering $\boldsymbol{C}_s$ and $\boldsymbol{C}_w$ to be a force $\boldsymbol{C}$, i.e., $\boldsymbol{C} = \boldsymbol{C}_s + \boldsymbol{C}_w$, the force balance diagram in Fig. 4(b) is simplified, as shown in Figs. 5(a) and (b).

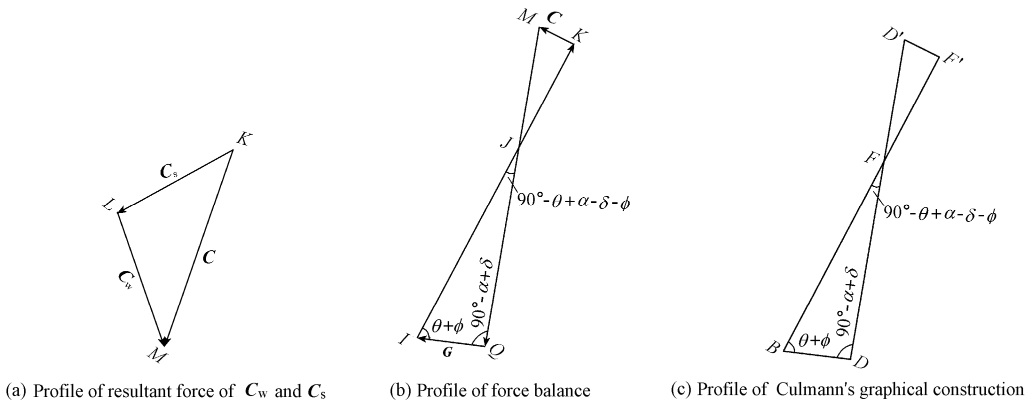


Fig. 5 Simplified force balance based on Culmann's method

Considering that

$$|\boldsymbol{C}_w| = c_w l_{AB} = \frac{c_w H}{\cos\alpha} \tag{7}$$

$$|\boldsymbol{C}_s| = c_s l_{BC} = c_s H \frac{\cos(\alpha - \beta)}{\cos\alpha \sin(\theta - \beta)} \tag{8}$$

Eq. (9) is obtained according to the law of cosines:

$$\left|\boldsymbol{C}\right|=\sqrt{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}-2\left|\boldsymbol{C}_{\mathrm{s}}\right|\left|\boldsymbol{C}_{\mathrm{w}}\right|\cos\left(90^{\circ}-\alpha+\theta\right)} \tag{9}$$

In ΔKLM (Fig. 5(a)), there is a definite solution to each angle when trilateral length is known, so instability would not occur. According to the law of cosines,

$$\angle MKL=\arccos\frac{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}\right|^{2}-\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}}{2\left|\boldsymbol{C}\right|\left|\boldsymbol{C}_{\mathrm{s}}\right|} \tag{10}$$

$$\angle MKJ=90^{\circ}+\phi-\angle MKL=90^{\circ}+\phi-\arccos\frac{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}\right|^{2}-\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}}{2\left|\boldsymbol{C}\right|\left|\boldsymbol{C}_{\mathrm{s}}\right|} \tag{11}$$

The relationship of the angles above was obtained from the force vector diagram. By analyzing ΔBDF in Fig. 2, the calculation diagram based on Culmann's method could be obtained, as shown in Fig. 5(c). In $\Delta FF'D'$, side $F'D'$, $\angle FF'D'$, $\angle F'FD'$, and $\angle FD'F'$ are known. Thus, this triangle is definite. BF', BD, $F'D'$, and DD' correspond to the soil reaction $\boldsymbol{R}$, the gravity on the sliding wedge $\boldsymbol{G}$, the resultant force $\boldsymbol{C}$, and the retaining wall reaction $\boldsymbol{E}$, respectively. Thus, the polygons in Figs. 5(b) and (c) are congruent. In Fig. 5(c), we can see

$$\angle BFD=90^{\circ}-\theta+\alpha-\delta-\phi \tag{12}$$

$$\angle D'F'B=\angle MKJ=90^{\circ}+\phi-\arccos\frac{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}\right|^{2}-\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}}{2\left|\boldsymbol{C}\right|\left|\boldsymbol{C}_{\mathrm{s}}\right|} \tag{13}$$

$l_{D'F}$ can be obtained by the law of sines:

$$\frac{l_{D'F}}{\sin\angle D'F'F}=\frac{l_{D'F'}}{\sin\angle D'FF'}=\frac{\left|\boldsymbol{C}\right|}{\sin\angle D'FF'} \tag{14}$$

$$l_{D'F}=\frac{\left|\boldsymbol{C}\right|\sin\angle D'F'F}{\sin\angle D'FF'}=\frac{\left|\boldsymbol{C}\right|\sin\left(90^{\circ}+\phi-\arccos\dfrac{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}\right|^{2}-\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}}{2\left|\boldsymbol{C}\right|\left|\boldsymbol{C}_{\mathrm{s}}\right|}\right)}{\sin\left(90^{\circ}-\theta+\alpha-\phi-\delta\right)} \tag{15}$$

$$l_{D'D}=l_{DF}+l_{D'F}=\frac{\left|\boldsymbol{G}\right|\sin(\theta+\phi)+\left|\boldsymbol{C}\right|\sin\left(90^{\circ}+\phi-\arccos\dfrac{\left|\boldsymbol{C}_{\mathrm{s}}\right|^{2}+\left|\boldsymbol{C}\right|^{2}-\left|\boldsymbol{C}_{\mathrm{w}}\right|^{2}}{2\left|\boldsymbol{C}\right|\left|\boldsymbol{C}_{\mathrm{s}}\right|}\right)}{\sin\left(90^{\circ}-\theta+\alpha-\delta-\phi\right)} \tag{16}$$

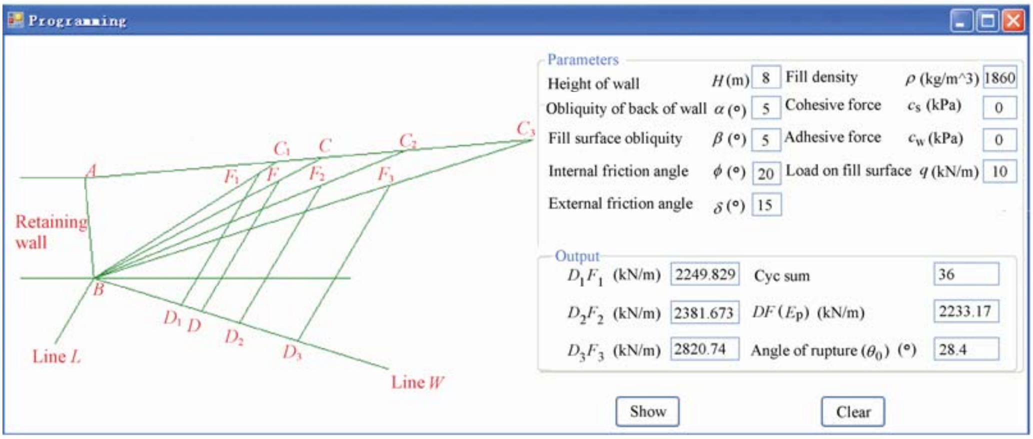
l_{DF}, which is obtained by Eq. (2), represents the retaining wall reaction without consideration of c_{s} and c_{w}. Eq. (16) is the formula for the passive earth pressure of cohesive soil. There is only one unknown parameter, θ, in Eq. (16). $l_{D'D}$, the magnitude of the retaining wall reaction of cohesive soil, can be obtained by programming. The minimum of $l_{D'D}$ is the passive earth pressure for cohesive soil, and the corresponding angle of rupture θ_0 can also be obtained.

If the backfill is heterogeneous soil, the gravity on the sliding wedge $\boldsymbol{G}$, the cohesive force on the sliding surface $\boldsymbol{C}_{\mathrm{s}}$, and the adhesive force on the interface of the retaining wall $\boldsymbol{C}_{\mathrm{w}}$ are expressed as functions of θ, based on the soil parameters. The formulas are more complex than those for homogeneous soil.

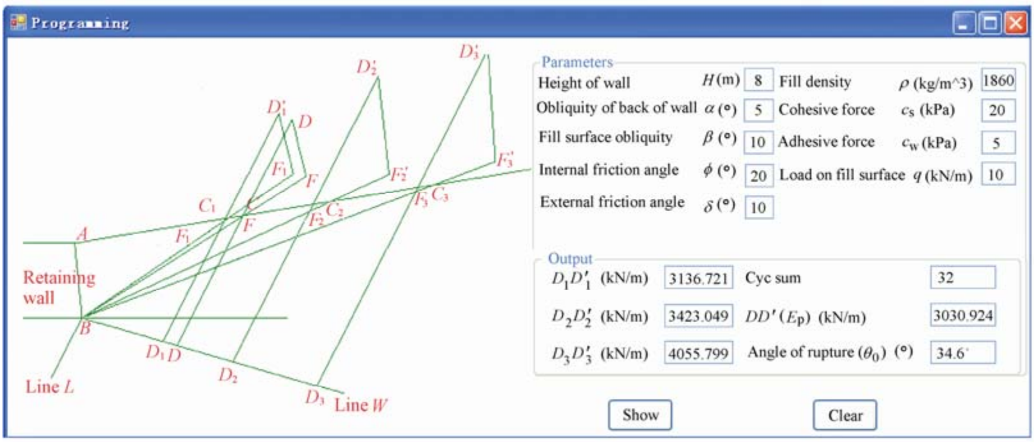
3 Programming and examples

A program was compiled with VB.NET programming software to calculate the passive earth pressure (Yuan 2009; Qi and Zhao 2009). The form of the fill surface in this program can be defined. If the fill surface is irregularly arc-shaped, some obvious inflection points can be taken to simulate the real situation. Then, the sliding wedge with loads in vertical direction can be calculated.

Sliding wedges ABC_i were taken as an example. Some parameters are shown in Fig. 6. The shape of the wedge was calculated by selecting a certain value of θ. Three sliding surfaces, BC_1, BC_2 and BC_3, were chosen randomly for comparison. Using the drawing function of the developed software, computational results were drawn at a certain scale (Fig. 6). DF, the minimum of D_iF_i in Fig. 6(a), demonstrates the passive earth pressure without consideration of c_s and c_w, and DD', the minimum of D_iD_i' in Fig. 6(b), demonstrates the passive earth pressure of cohesive soil. This example indicates that the program is effective, universal, and convenient. It can trace and export visualized dynamic information through graphics.



(a) Without consideration of cohesive force and adhesive force



(b) With consideration of cohesive force and adhesive force (case 6 of example 1)

Fig. 6 Output interface of computation for regular fill surface

Example 1: A cohesive soil sliding wedge with an extra load on the fill surface was studied. The retaining wall height H was 8 m. Fill parameters were $\rho = 1\ 860$ kg/m^3 and $\phi = 20°$. The load on the fill surface was $q = 10$ kN/m. Other related parameters and calculated results are shown in Table 1. Calculated results from Hu and Tan (2009) and the Coulumb earth pressure and Rankine earth pressure formulas are also shown in Table 1 for comparison.

Table 1 Calculated results of example 1

Case	α(°)	β(°)	δ(°)	c_s (kPa)	c_w (kPa)	Calculated result of present method		Calculated result of $\|E_p\|$ with other methods (kN/m)		
						$\|E_p\|$ (kN/m)	θ_0(°)	Hu and Tan (2009)	Rankine theory	Coulumb theory
1	0	0	0	0	0	1 377.1	32.6	1 377.1	1 377.1	1 377.1
2	0	0	0	10	0	1 605.6	32.6	1 605.6	1 605.6	NA
3	5	5	5	0	0	1 675.2	37.6	1 675.1	NA	1 676.4
4	5	5	15	0	0	2 233.2	28.4	2 233.4	NA	2 235.1
5	5	10	10	20	0	2 962.3	37.4	2 963.1	NA	NA
6	5	10	10	20	5	3 030.9	34.6	3 030.5	NA	NA
7	5	10	10	20	10	3 097.3	34.6	3 097.9	NA	NA
8	5	10	10	20	15	3 162.9	34.6	3 162.7	NA	NA

Note: NA means inapplicability.

Example 2: A non-cohesive soil sliding wedge without extra loads on the fill surface was studied. $l_{AA'}$, the distance between the wall top and the fill surface turning point A', was 5 m. The angle β between AA' and the horizontal plane was 18°. The part of the fill surface ($A'C$) was horizontal. Other parameters and the state of the fill surface are shown in Fig. 7. The magnitude of the passive earth pressure $\boldsymbol{E}_{\mathrm{p}}$ was 2 309.6 kN/m and the angle of rupture θ_0 was 27.2° (Fig. 7).

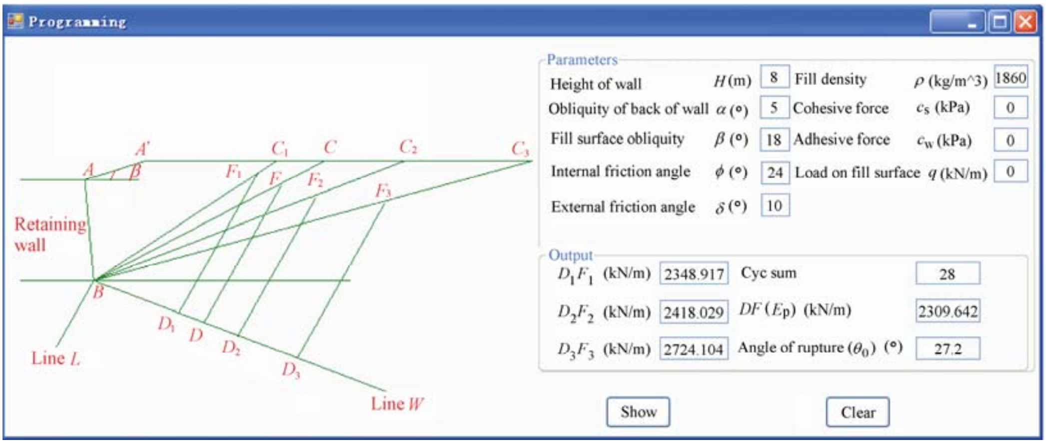


Fig. 7 Output interface of computation for irregular fill surface (result of example 2)

The calculated results of example 1 in this study basically agree with the results of Hu and Tan (2009) and the calculated results from the Rankine theory and Coulumb theory formulas. Thus, this formula is in accordance with the classic Rankine formula and Coulumb formula under the Rankine or Coulomb assumptions. Moreover, the sliding surface obliquity can be obtained. When considering the cohesive force on the sliding surface $\boldsymbol{C}_{\mathrm{s}}$ and the adhesive force on the interface of the retaining wall $\boldsymbol{C}_{\mathrm{w}}$, the results are quite different. Example 2 indicates that Culmann's graphical construction is superior. It is applicable to the irregular fill surface as well.

4 Conclusions

Based on the sliding plane hypothesis of Coulumb earth pressure theory, a new method for calculation of the passive earth pressure of cohesive fill was constructed with Culmann's graphical construction. The influence of the cohesive force, adhesive force, and the fill surface form can be considered in this method. The results of examples obtained by this method were consistent with the results from the formulas of Rankine and Coulumb earth pressure theories under their assumed conditions. Moreover, in contrast to the classical Rankine and Coulumb earth pressure theories, this method can be used under the conditions of cohesive fill, irregular fill surfaces, and a fill surface with loads. It can also be used under the condition of non-cohesive fill. This method is conceptually clear, and the corresponding formulas given in this paper are simple and convenient for application.

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