Water Science and Engineering, 2011, 4(3): 305-316
doi:10.3882/j.issn.1674-2370.2011.03.007

http://www.waterjournal.cn
e-mail: wse2008@vip.163.com

Implementation and verification of interface constitutive model in FLAC^{3D}

Hai-min WU*[1], Yi-ming SHU[1], Jun-gao ZHU[2]

1. College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, P. R. China
2. Institute of Geotechnical Engineering, Hohai University, Nanjing 210098, P. R. China

Abstract: Due to the complexity of soil-structure interaction, simple constitutive models typically used for interface elements in general computer programs cannot satisfy the requirements of discontinuous deformation analysis of structures that contain different interfaces. In order to simulate the strain-softening characteristics of interfaces, a nonlinear strain-softening interface constitutive model was incorporated into fast Lagrange analysis of continua in three dimensions (FLAC^{3D}) through a user-defined program in the FISH environment. A numerical simulation of a direct shear test for geosynthetic interfaces was conducted to verify that the interface model was implemented correctly. Results of the numerical tests show good agreement with the results obtained from theoretical calculations, indicating that the model incorporated into FLAC^{3D} can simulate the nonlinear strain-softening behavior of interfaces involving geosynthetic materials. The results confirmed the validity and reliability of the improved interface model. The procedure and method of implementing an interface constitutive model into a commercial computer program also provide a reference for implementation of a new interface constitutive model in FLAC^{3D}.
Key words: *interface element; constitutive model; FLAC^{3D}; programming in FISH environment*

1 Introduction

Modeling of soil-structure interaction is very important in geotechnical engineering including hydraulic structures. It is relevant to a wide range of project problems, such as numerical analysis of concrete-faced rockfill dams, retaining walls, shallow foundations, piles, tunnels, reinforced earthworks, and geosynthetic liners. The relationship between shear stress and shear displacement of the soil-structure interface plays a major role in modeling soil-structure interactions. Through extensive interface shear testing, many investigators have found that the shear stress-displacement behavior of different types of interfaces can vary significantly. Based on the results of experiments, researchers have proposed several types of interface constitutive models, including the nonlinear elastic model (Clough and Duncan 1971), elastic-perfectly plastic model (Brandt 1986; Zhou and Lu 2009), nonlinear elastic-perfectly plastic model (Luan and Wu 2004), rigid-plastic model (Yin et al. 1995), elastic-viscoplastic

This work was supported by the National Natural Science Foundation of China (Grant No. 51079047).
*Corresponding author (e-mail: *haimin-wu@hotmail.com*)
Received Jan. 8, 2011; accepted Apr. 18, 2011

model (Qian and Zhan 1993), damage model (Hu and Pu 2003), strain-softening model (Esterhuizen et al. 2001, Kim 2007), monotonic and cyclic model (Zhang et al. 2008), and cracking model (Caballero et al. 2008). Whereas there are many available interface models, most numerical analysis softwares typically only provide a single and simple generalized interface model for users. This seriously limits the application of these softwares in solving problems that involve different soil-structure interfaces.

Due to its distinct advantage of solving large strain geotechnical deformation problems, fast Lagrange analysis of continua in three dimensions (FLAC^{3D}) has been widely used in fields of hydraulic, geotechnical, mining, geological, and port and waterway engineering since the 1990s. FLAC^{3D} provides many built-in constitutive models that can be employed to simulate the complicated mechanical behaviors of different kinds of soils and rocks. It also offers a development platform that enables users to implement user-defined models and other functions. There are two methods to implement a user-defined model through the development platform of FLAC^{3D}. The first method is to implement the model through VC++ programming. Using this method, Chu et al. (2006) implemented a viscoelasto-plastic rheological constitutive model for simulating the mechanical behavior of rocks, and Chen and Liu (2007) implemented a Duncan-Chang constitutive model to simulate the nonlinear property of soil. Another method is to implement the model using the embedded language (FISH) of FLAC^{3D}. Many investigators have successfully defined their own variables and functions to model different mechanical soil-structural behaviors with this method. For example, Qi et al. (2004) modified the model of bolt elements with the FISH program in FLAC^{2D} in order to simulate the damaging mechanism of rock bolts; Yao et al. (2007) programmed a FISH function to automatically solve the bending moment, axial force, and safety factor of strength of liner elements; Xu et al. (2008) modified the geogrid elements with FISH to model the direct and pullout tests of the geomembrane-sand interface. These investigations on further development have not only extended many functions of the software but have also gathered an abundance of experience for other developers.

FLAC^{3D} provides interfaces that are characterized by Coulomb sliding and/or tensile and shear bonding. Interfaces have the properties of friction, cohesion, dilation, normal and shear stiffnesses, and tensile and shear bond strengths. The built-in interface element in FLAC^{3D} can only simulate the relationship between shear stress and shear displacement according to the linear elastic-perfectly plastic model. But it cannot be used to simulate interfaces characterized by nonlinear and strain-softening behavior. This restricts the application of FLAC^{3D} in numerical analysis of soil-structure interaction problems. Studies on implementation and further development of interface elements in FLAC^{3D} have been rarely reported. Zhang and Xu (2005) improved the normal calculation method of interface elements in FLAC^{3D} to simulate the mechanical behavior of a joint with initial width. However, no improved constitutive relationships for the interface element have been implemented. As a result, many types of relationships between shear stress and displacement of the interface cannot be simulated by FLAC^{3D}.

In this study, the linear elastic-perfectly plastic constitutive relationship between shear stress and shear displacement of interface elements in FLAC^{3D} was improved. The linear elastic portion of the shear stress-shear displacement relationship was replaced by a nonlinear (hyperbolic) elastic relationship originally developed by Clough and Duncan (1971). The perfectly plastic portion was replaced by a nonlinear strain-softening model developed by Esterhuizen et al. (2001) for simulation of the nonlinear strain-softening behavior of geosynthetic interfaces after the displacement reaches its peak strength. A hypothetical problem of direct shearing on a geomembrane-geotextile interface was solved both numerically and theoretically, and the results of the numerical computations were compared with the theoretical results to verify the newly improved model.

2 Constitutive model of interface element in FLAC^{3D}

Interfaces in FLAC^{3D} are collections of triangular interface elements, each of which is defined by three interface nodes. Interface elements are attached to a zone boundary through interface nodes. As shown in Fig. 1, the constitutive model is defined by a linear elastic-perfectly plastic Coulomb shear strength criterion that limits the shear force acting at an interface node, normal and shear stiffnesses (k_n and k_s), tensile and shear bond strengths (σ_t and S_s), and a dilation angle (ψ) that causes an increase in effective normal force on the target face after the shear-strength limit is reached (Itasca Consulting Group 2005). The relationship between shear stress and shear displacement is illustrated in Fig. 2; the curve includes two parts: the linear elastic stage and the perfectly plastic stage.

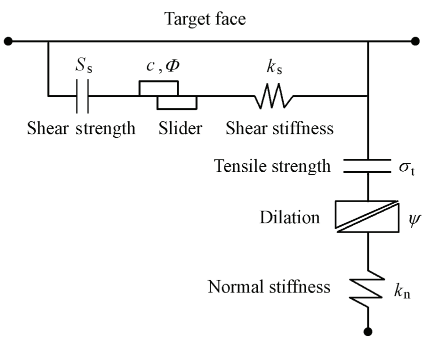

Fig. 1 Computation theory of interface element

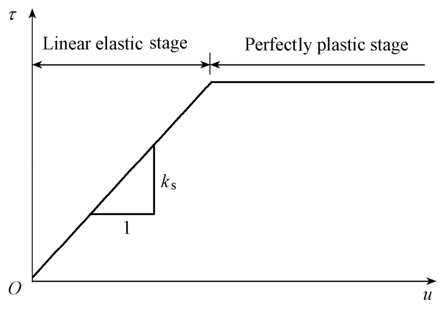

Fig. 2 Relationship between shear stress τ and shear displacement u of interface element

2.1 Linear elastic stage

In the linear elastic stage, the normal and shear forces at the calculation time $t+\Delta t$ are determined by Eqs. (1) and (2):

$$F_{\mathrm{n}}^{(t+\Delta t)} = k_{\mathrm{n}} u_{\mathrm{n}} A + \sigma_{\mathrm{n}0} A \tag{1}$$

$$F_{\mathrm{si}}^{(t+\Delta t)} = F_{\mathrm{si}}^{(t)} + k_{\mathrm{s}} \Delta u_{\mathrm{si}}^{(t+\Delta t/2)} A + \sigma_{\mathrm{si}} A \tag{2}$$

where $F_{\mathrm{n}}^{(t+\Delta t)}$ is the normal force at the calculation time $t+\Delta t$; k_{n} is the normal stiffness of interface element; u_{n} is the normal penetration of the interface node into the target face; $\sigma_{\mathrm{n}0}$ is the normal effective stress added due to interface stress initialization; A is the representative area associated with the interface node; $F_{\mathrm{si}}^{(t+\Delta t)}$ and $F_{\mathrm{si}}^{(t)}$ are the shear forces at calculation times $t+\Delta t$ and t, respectively; k_{s} is the shear stiffness of interface element, which is a constant in the elastic stage; $\Delta u_{\mathrm{si}}^{(t+\Delta t/2)}$ is the incremental shear displacement between t and $t+\Delta t$; and σ_{si} is the vector of additional shear stress due to interface stress initialization.

2.2 Perfectly plastic stage

According to the Mohr-Coulomb criteria, the contact states of the interface node can be divided into three types: the slip while bonded state, the Coulomb sliding state, and the separated state. The yield relationships in the shear and normal directions are

$$F_{\mathrm{s\,max}} = cA + \tan\Phi\left(F_{\mathrm{n}} - pA\right) \tag{3}$$

$$F_{\mathrm{n}} = \sigma_{\mathrm{t}} \tag{4}$$

where $F_{\mathrm{s\,max}}$ is the maximum shear strength, c is the cohesion of the interface, Φ is the friction angle of the interface, F_{n} is the normal force, p is the pore pressure, and σ_{t} is the normal tensile strength of the interface.

At every calculation time *t*, the normal force F_{n} and shear force F_{si} of interface nodes are compared with the normal tensile strength σ_{t} and maximum shear strength $F_{\mathrm{s\,max}}$, respectively. If the interface force exceeds the interface strength, the force will be corrected as follows (Itasca Consulting Group 2005):

(1) If $F_{\mathrm{n}} < \sigma_{\mathrm{t}}$ and $F_{\mathrm{si}} < F_{\mathrm{s\,max}}$, the interface node remains in the elastic stage, falling into the slip while bonded state. The interface forces do not need to be corrected.

(2) If $F_{\mathrm{n}} < \sigma_{\mathrm{t}}$ and $F_{\mathrm{si}} \geq F_{\mathrm{s\,max}}$, the interface node falls into the Coulomb sliding state. The interface forces need to be corrected.

If the interface does not have dilation characteristics, the forces are corrected as follows:

$$F_{\mathrm{si}} = F_{\mathrm{s\,max}} \tag{5}$$

If the interface has dilation characteristics, the forces are corrected as follows:

$$F_{\mathrm{n}} = F_{\mathrm{n}} + \frac{\left(\left|F_{\mathrm{s}}\right|_0 - F_{\mathrm{s\,max}}\right)}{Ak_{\mathrm{s}}} \tan\psi k_{\mathrm{n}}, \quad F_{\mathrm{si}} = F_{\mathrm{s\,max}} \tag{6}$$

where $\left|F_{\mathrm{s}}\right|_0$ is the shear force before the above corrections are made, and ψ is the dilation angle of the interface.

(3) If $F_{\mathrm{n}} \geq \sigma_{\mathrm{t}}$, the interface node falls into the separated state, and the interface forces are corrected as follows:

$$F_{\mathrm{n}} = 0,\ F_{\mathrm{si}} = 0,\ \sigma_{\mathrm{t}} = 0 \tag{7}$$

3 Improvement of interface constitutive model in FLAC^{3D}

Composite barrier systems on the slope of an impoundment or landfill are usually composed of different geosynthetic and soil materials (Wu et al. 2008; Fowmes et al. 2008). Due to low shearing resistance, the geosynthetic interface usually becomes the preferential slipping surface of the composite barriers. Geosynthetic interfaces are susceptible to strain softening: they exhibit a decrease in shear stress at displacements beyond the corresponding displacement of peak strength (Byrne 1994; Kim 2007; Anubhav and Basudhar 2010). The deformation of barriers of impoundments or landfills resulted from the water pressure or waste settlements may induce large shear displacement at the geosynthetic-soil or geosynthetic-geosynthetic interfaces (Filz et al. 2001). Under conditions of large shear displacement, the displacement-softening characteristics of geosynthetic interfaces become the controlling consideration in the analysis of stability and deformation of different components. Based on the results of interface tests conducted by Jones and Dixon (1998), a new interface constitutive model that combines a nonlinear hyperbolic model (Clough and Duncan 1971) with a displacement-softening model (Esterhuizen et al. 2001) was proposed to describe the nonlinear strain-softening behavior of geosynthetic interfaces. The detailed constitutive relationships and procedures of implementation in FLAC^{3D} of the new interface model are described in the following sections.

3.1 Nonlinear strain-softening interface model

As shown in Fig. 3, the typical shear stress versus displacement behavior of a textured geomembrane-geotextile interface was derived from a test carried out using a 300-mm direct shear apparatus as described by Jones and Dixon (1998). The nonlinear strain-softening property of geosynthetic interfaces exhibits a reduction in shear stress at displacements beyond the corresponding displacement of peak strength. The shear stress-displacement relationship can be described by three stages: the pre-peak elastic stage, the softening stage, and the residual stage. In the pre-peak stage, the shear stress increases from the origin with increasing displacement and typically follows a nonlinear stress-displacement behavior until a peak value is reached. In the softening stage, with the increase of shear displacement, there appears a reduction in shear stress until a constant or residual value is reached. In the residual stage, the shear stress remains constant when the shear displacement increases. The nonlinear and strain-softening behaviors of geosynthetic interfaces can be described by the new interface model, which combines the nonlinear hyperbolic model with the displacement-softening model.

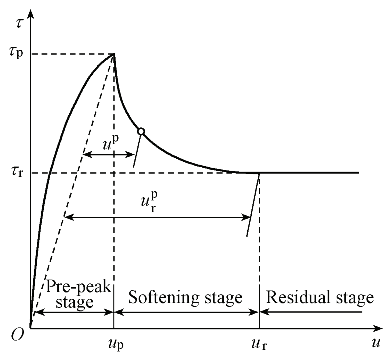

Fig. 3 Relationship between shear stress and shear displacement of geosynthetic interface

3.1.1 Pre-peak stage

As shown in Fig. 3, the relationship between interface shear stress and displacement in the pre-peak stage can typically be modeled by a hyperbolic shape. The pre-peak interface behavior can be represented by the nonlinear hyperbolic model (Clough and Duncan 1971). The shear stiffness of the interface k_{s} can be expressed as

$$k_{\mathrm{s}} = K_{\mathrm{i}}\gamma_{\mathrm{w}}\left(\frac{\sigma_{\mathrm{n}}}{P_{\mathrm{a}}}\right)^{n}\left(1-R_{\mathrm{f}}\frac{\tau}{c_{\mathrm{p}}+\sigma_{\mathrm{n}}\tan\Phi_{\mathrm{p}}}\right)^{2} \tag{8}$$

where $\gamma_{\mathrm{w}} = \rho_{\mathrm{w}} g$, where ρ_{w} is the density of water, g is the acceleration of gravity; σ_{n} is the normal effective stress of the interface; P_{a} is the atmospheric pressure; τ is the shear stress; c_{p} is the peak cohesion of the interface; Φ_{p} is the peak friction angle of the interface; and K_{i}, R_{f}, and n are the nonlinear parameters that can be derived from interface direct shear tests.

3.1.2 Softening stage

In the softening stage, the shear strength initially shows a sharp reduction with the shear displacement, and then shows a gradual strength reduction. The shear stress-displacement relationship can be represented by the displacement-softening model proposed by Esterhuizen et al. (2001). The strength residual factor R can be defined as

$$R = \frac{\tau_{\mathrm{p}}-\tau_{\mathrm{pr}}}{\tau_{\mathrm{p}}-\tau_{\mathrm{r}}} \tag{9}$$

where τ_{pr} is the post-peak shear strength, τ_{p} is the peak shear strength, and τ_{r} is the residual shear strength. The parameters, τ_{p} and τ_{r}, can be calculated by the Mohr-Coulomb criterion using Eqs. (10) and (11):

$$\tau_{\mathrm{p}} = \sigma_{\mathrm{n}}\tan\Phi_{\mathrm{p}} + c_{\mathrm{p}} \tag{10}$$

$$\tau_{\mathrm{r}} = \sigma_{\mathrm{n}}\tan\Phi_{\mathrm{r}} + c_{\mathrm{r}} \tag{11}$$

where Φ_{r} is the residual friction angle of the interface, and c_{r} is the residual cohesion of the interface.

The shear displacement ratio D is expressed as

$$D = u^{\mathrm{p}}/u_{\mathrm{r}}^{\mathrm{p}} \tag{12}$$

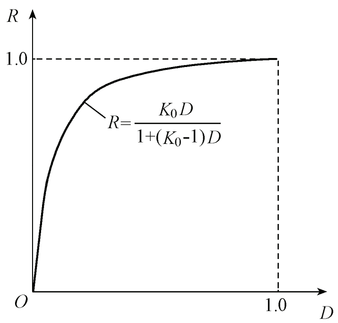

Fig. 4 Typical *R-D* curve

where u^{p} is the plastic shear displacement (Fig. 3), and $u_{\mathrm{r}}^{\mathrm{p}}$ is the plastic shear displacement where the shear stress just reaches the residual strength. The results of direct shear tests conducted by Shallenberger and Filz (1996) indicate that $u_{\mathrm{r}}^{\mathrm{p}}$ is a constant under the same normal pressure.

The typical *R-D* curve is shown in Fig. 4. The nonlinear curve can be approximated by a single hyperbolic relationship between the strength residual factor R and the shear displacement ratio D. The relationship is expressed by Eq. (13):

$$R = \frac{K_0 D}{1 + (K_0 - 1)D} \tag{13}$$

where K_0 is the initial slope of the *R*-*D* curve.

3.1.3 Residual stage

After reaching the residual stage, the shear stress reaches a stable state with the continuous increasing of shear displacement as shown in Fig. 3. The shear stress stays at a constant value equal to the residual shear strength τ_{r}.

3.2 Implementation of nonlinear strain-softening interface model

The constitutive relation in three stages of the nonlinear strain-softening interface model was incorporated into FLAC^{3D} by the user-defined FISH program. The general process of implementation is as follows:

At every calculation step, the program first read the normal effective stress, shear stress, and shear displacement of every interface element. The state of every interface element was judged by the yield criterion according to the shear stress and shear displacement. According to the state of the interface element, constitutive relations were selected to calculate the relevant stiffness and strength parameters using the interface input parameters. Then, the calculated parameters for each interface element were used to carry out the calculation of the next step. In this way, the program continuously circulated until all elements reached an equilibrium state. The detailed program flowchart is shown in Fig. 5.

Due to the limit of length for this paper, the source code of the program cannot be presented in detail, but several key points about the implementation procedure are described here:

(1) At every calculation step, it is checked whether the interface normal effective stress σ_{n} read from the last step is positive. In the beginning of the computation cycle, the stress and displacement of the elements are still in an unbalanced state and the normal effective stress read from the last step may be a negative number or zero; this will result in an error of the program. In this case, a small positive number should be input as the initial value of σ_{n}.

(2) The shear stress and shear displacement are not variables of the interface element, but they are variables at the interface nodes in FLAC^{3D}. Since the determination of the interface state (elastic, plastic, etc.) is based on the shear stress and shear displacement of the interface element, the nodal shear stress and shear displacement obtained from the last step must be transformed into variables of the element before calculation of the new interface parameters.

(3) In the softening and residual stages, only the reduced strength parameters need to be entered. After the reduction of strength, the plastic flow occurs, interface elements begin to yield, and the shear stress will be corrected according to the original yield criteria (Eqs. (3) through (7)) of FLAC^{3D}. It is not the shear stiffness but the shear strength parameters that will

control the computation. It must be clarified that the method described in this paper mainly deals with the implementation of a constitutive model of interface elements for simulation of the nonlinear strain-softening behavior of the soil-structure interface. For the method of improving the normal constitutive relations of interface elements in FLAC^{3D}, the reader is referred to the research conducted by Zhang and Xu (2005).

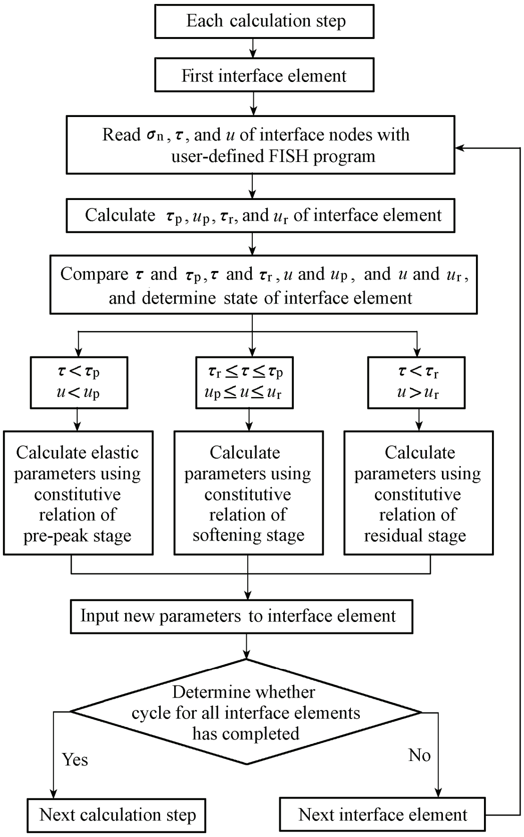

Fig. 5 Calculation flowchart of improved constitutive model for interface element

4 Model verification

4.1 Numerical test

In order to verify the effectiveness of the improved model, a simple numerical example was used to model the interface direct shear test between a geomembrane and a geotextile. As shown in Fig. 6, the numerical model of the test was composed of two parts: the upper part was a shear box with soil in it, and the lower part was a soil block where the geomembrane was glued to the top. The geotextile was fixed on the bottom surface of the upper box. In order to maintain a constant contact area during shearing, the surface of the lower box was larger than that of the upper one. The improved nonlinear strain-softening model was used to simulate the geomembrane-geotextile interface. The normal behavior was modeled using the initial contact relation provided by FLAC^{3D} (Eq. (1)). The normal stiffness was calculated by Eq. (15):

$$k_{\mathrm{n}} = 10\max\left[\frac{(3K+4G)}{3Z_{\min}}\right] \tag{15}$$

where K and G are the bulk modulus and shear modulus of the adjoining zones on the two sides of the interface, respectively; and $Z_{\min}$ is the minimum width of the adjoining zones on both sides of the interface.

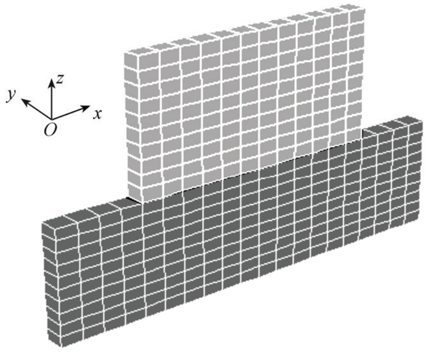

Fig. 6 Grid of numerical test model

In order to compare the computational results with theoretical results, a linear elastic model was employed for the soil in the upper as well as the lower boxes. Gravity forces were not considered during the numerical experiment. Parameters for the improved interface model were determined from the interface direct shear tests conducted by Jones and Dixon (1998). The parameters are listed in Table 1.

Table 1 Parameters of strain-softening model of interface

K_{i}	n	R_{f}	c_{p} (Pa)	Φ_{p} (°)	c_{r} (Pa)	Φ_{r} (°)	K_0	$u_{\mathrm{r}}^{\mathrm{p}}$ (m)	σ_{t} (Pa)	ψ (°)	k_{n} (kN/m^3)
4550	0.33	0.825	2 500	23	700	11.7	10	0.08	0	0	10^9

According to the typical procedures for direct shear test, a constant normal pressure was applied on the top surface of the soil. Then, the displacements and velocities of all elements were reset to zero. A fixed horizontal velocity of 5×10^{-5} m/s was applied to all the elements of the upper part to simulate the actual shearing rate of 3 mm/min (Jones and Dixon 1998). This led to a displacement on the interface between the upper soil and the lower block. Four numerical direct shear tests were simulated with constant normal pressures of 50 kPa, 100 kPa, 150 kPa, and 200 kPa, respectively.

4.2 Results and analysis

The relationships between the average shear stress and shear displacement of the interface under different normal pressures are shown in Fig. 7. The numerical results show good agreement with theoretical curves calculated by Eqs. (8) through (14) using the same parameters. Fig. 7 illustrates that the improved interface model in FLAC^{3D} is capable of modeling the nonlinear pre-peak behavior as well as the strain-softening post-peak response.

In order to confirm that the improved interface model can reflect the mechanical mechanism of the direct test and the failure process of the interface elements, the distribution of interface shear stress and normal effective stress at different shear displacements are shown in Fig. 8 and Fig. 9, respectively.

As shown in Fig. 8, the normal effective stresses at different shear displacements are uniformly distributed in most parts of the interface except at the two sides. The normal effective stress on the left side is lower than that in the middle part. The normal effective stress on the right side is larger than that in the middle part. This is because the shearing starts from

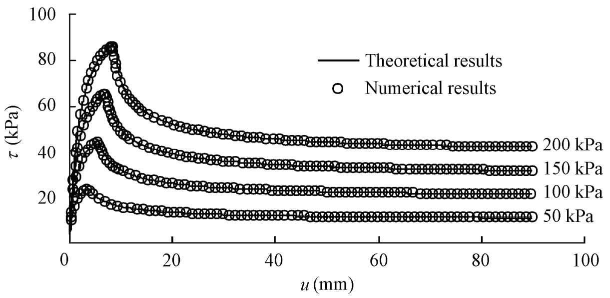

Fig. 7 Relationship between shear stress and shear displacement of interface

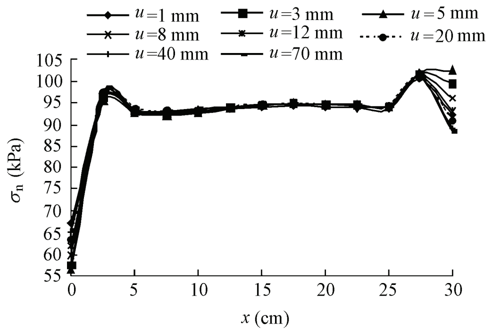

Fig. 8 Distribution of interface normal effective stress at different shear displacements (σ_n = 100 kPa)

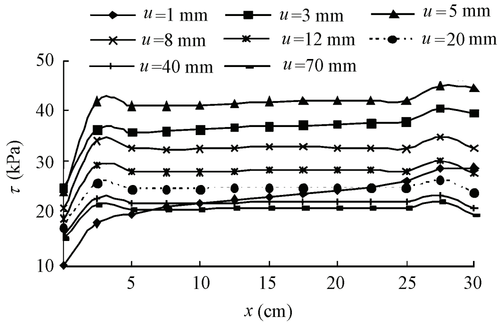

Fig. 9 Distribution of interface shear stress at different shear displacements (σ_n = 100 kPa)

left to right. It can also be seen that the normal effective stresses in the middle part and on the left side do not change at different shear displacement values.

It can be seen from Fig. 9 that the shear stress increases with the shear displacement, but decreases when the shear displacement exceeds 5 mm. After the shear displacement exceeds 40 mm, the shear stress does not change significantly and gradually reaches a stable value. This suggests that the interface elements reach the strain-softening stage when the shear displacement exceeds 5 mm, and are close to the residual stage when the shear displacement exceeds 40 mm under a normal pressure of 100 kPa. This presents a consistency with the shear stress-displacement curve (Fig. 7), which is close to the peak state at a displacement of 5.28 mm, and the residual state at a displacement of 40 mm under a normal pressure of 100 kPa. Besides, the distribution of shear stress along the middle part of the interface gradually becomes uniform when the shear displacement exceeds 5 mm. This is because the plastic flow occurs in most of the interface elements after they reach the softening stage, and the upper elements begin sliding along the interface with a similar velocity.

5 Conclusions

(1) The interface elements in FLAC^{3D} can only be used to simulate soil and structure interfaces that accord with the linear elastic-perfectly plastic model. This restricts the application of FLAC^{3D} when performing numerical analysis of geotechnical problems involving various types of interfaces. In this study, a nonlinear strain-softening model was incorporated into FLAC^{3D} to simulate the nonlinear and strain-softening behavior of geosynthetic interfaces. This model presents a significant improvement over the existing built-in linear elastic-perfectly plastic model for interface elements in FLAC^{3D}, when modeling geosynthetic interfaces.

(2) A numerical simulation of direct shear test of a geomembrane-geotextile interface was performed to verify the improved interface model. The numerical results present good agreement with the theoretical solution. The new interface model incorporated into FLAC^{3D} can reflect the mechanical mechanism of direct shear testing and the failure processes of the interface elements.

(3) The method and basic procedures described in this paper detail the implementation of a constitutive model for interface elements in FLAC^{3D} to simulate the nonlinear and strain-softening behavior of soil-structure interfaces. They also offer a reference for incorporation of other interface constitutive models into FLAC^{3D} using the FISH programming platform.

References

Anubhav, and Basudhar, P. K. 2010. Modeling of soil-woven geotextile interface behavior from direct shear test results. *Geotextiles and Geomembranes*, 28(4), 403-408. [doi:10.1016/j.geotexmem.2009.12.005]

Brandt, J. R. T. 1986. *Behavior of Soil Concrete Interface*. Ph. D. Dissertation. Edmonton: The University of Alberta.

Byrne, R. J. 1994. Design issues with strain-softening interfaces in landfill liners. *Proceedings of the Waste Technology*, 1-26. Charleston.

Caballero, A., Willam, K. J., and Carolc, I. 2008. Consistent tangent formulation for 3D interface modeling of cracking/fracture in quasi-brittle materials. *Computer Methods in Applied Mechanics and Engineering*, 197(33-40), 2804-2822. [doi:10.1016/j.cma.2008.01.011]

Chen, Y. M., and Liu, H. L. 2007. Development and implementation of Duncan-Chang constitutive model in FLAC^{3D}. *Rock and Soil Mechanics*, 28(10), 2123-2126. (in Chinese)

Chu, W. J., Xu, W. Y., Yang, S. Q., and Zhou, W. Y. 2006. Secondary development of a viscoelasto-plastic rheological constitutive model of rock based on FLAC^{3D}. *Rock and Soil Mechanics*, 27(11), 2005-2010. (in Chinese)

Clough, G. W., and Duncan, J. M. 1971. Finite element analysis of retaining wall behavior. *Journal of Soil Mechanics and Foundation Engineering*, 97(12), 1657-1672.

Esterhuizen, J. J. B., Filz, G. M., and Duncan, J. M. 2001. Constitutive behavior of geosynthetic interfaces. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(10), 834-840. [doi:10.1061/(ASCE)

1090-0241(2001)127:10(834)]
Filz, G. M., Esterhuizen, J. J. B., and Duncan, J. M. 2001. Progressive failure of lined waste impoundments. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(10), 841-848. [doi:10.1061/(ASCE) 1090-0241(2001)127:10(841)]
Fowmes, G. J., Dixon, N., and Jones, D. R. V. 2008. Validation of a numerical modelling technique for multilayered geosynthetic landfill lining systems. *Geotextiles and Geomembranes*, 26(2), 109-121. [doi: 10.1016/j.geotexmem.2007.09.003]
Hu, L., and Pu, J. L. 2003. Application of damage model for soil-structure interface. *Computers and Geotechnics*, 30(2), 165-183. [doi:10.1016/S0266-352X(02)00059-9]
Itasca Consulting Group. 2005. *Fast Lagrangian Analysis of Continua in 3 Dimensions*. Minneapolis: Itasca Consulting Group.
Jones, D. R. V., and Dixon, N. 1998. Shear strength properties of geomembrane-geotextile interfaces. *Geotextiles and Geomembranes*, 19(1), 45-57. [doi:10.1016/S0266-1144(00)00002-9]
Kim, D. 2007. *Multi-Scale Assessment of Geotextile-Geomembrane Interaction*. Ph. D. Dissertation. Atlanta: Georgia Institute of Technology.
Luan, M. T, and Wu, Y. J. 2004. A nonlinear elasto-perfectly plastic model of interface element for soil-structure interaction and its applications. *Rock and Soil Mechanics*, 25(4), 507-513. (in Chinese)
Qi, T. Y., Lu, S. L., and Gao, B. 2004. Amended model of bolt element in FLAC and its application. *Chinese Journal of Rock Mechanics and Engineering*, 23(13), 2197-2200. (in Chinese)
Qian, J. H., and Zhan, M. L. 1993. Tests and analysis on rheological characteristic of interface. *Proceedings of Research on Interaction Between Soil and Hydraulic Structures*. Beijing: China WaterPower Press. (in Chinese)
Shallenberger, W. C., and Filz, G. M. 1996. Interface strength determination using a large displacement shear box. *Proceedings of Second International Conference on Environmental Geotechnics*, 147-152. Osaka.
Wu, W., Wang, X. T., and Aschauer, F. 2008. Investigation on failure of a geosynthetic lined reservoir. *Geotextiles and Geomembranes*, 26(4), 363-370. [doi:10.1016/j.geotexmem.2007.12.001]
Xu, C., Liao, X. Y., and Ye, G. B. 2008. Numerical analysis on strain-softening characteristic of soil-geosynthetic interface. *Chinese Journal of Hunan University (Natural Sciences)*, 35(11), 177-181. (in Chinese)
Yao, C. C., Yu, H. L., and Zhang, Y. 2007. Method of calculating safety factor of liner based on FLAC^{3D} software entity. *Sichuan Architecture*, 27(2), 115-116. (in Chinese)
Yin, Z. Z., Zhu, H., and Xu, G. H. 1995. A study of deformation in the interface between soil and concrete. *Computers and Geotechnics*, 17(1), 75-92. [doi:10.1016/0266-352X(95)91303-L]
Zhang, G., Wang, L. P., and Zhang, J. M. 2008. Unified modeling of monotonic and cyclic behavior of interface between structure and gravelly soil. *Soils and Foundations*, 48(2), 231-245. [doi:10. 1002/nag.865]
Zhang, G. K., and Xu, W. Y. 2005. Numerical simulation of discontinuous contact behavior of vertical construction joints in gravity dam. *Journal of Hydraulic Engineering*, 36(8), 982-987. (in Chinese)
Zhou, A. Z., and Lu, T. H. 2009. Elasto-plastic constitutive model of soil-structure interface in consideration of strain softening and dilation. *Acta Mechanica Solida Sinica*, 22(2), 171-179. [doi:10.1016/S0894-9166(09)60102-6]