

Application of thermodynamics-based rate-dependent constitutive models of concrete in the seismic analysis of concrete dams

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Abstract: This paper discusses the seismic analysis of concrete dams with consideration of material nonlinearity. Based on a consistent rate-dependent model and two thermodynamics-based models, two thermodynamics-based rate-dependent constitutive models were developed with consideration of the influence of the strain rate. They can describe the dynamic behavior of concrete and be applied to nonlinear seismic analysis of concrete dams taking into account the rate sensitivity of concrete. With the two models, a nonlinear analysis of the seismic response of the Koyna Gravity Dam and the Dagangshan Arch Dam was conducted. The results were compared with those of a linear elastic model and two rate-independent thermodynamics-based constitutive models, and the influences of constitutive models and strain rate on the seismic response of concrete dams were discussed. It can be concluded from the analysis that, during seismic response, the tensile stress is the control stress in the design and seismic safety evaluation of concrete dams. In different models, the plastic strain and plastic strain rate of concrete dams show a similar distribution. When the influence of the strain rate is considered, the maximum plastic strain and plastic strain rate decrease.

Key words: concrete; constitutive model; rate dependency; concrete dam; nonlinearity; seismic analysis

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1 Introduction

China has abundant water resources. As part of the national energy and water conservancy plan, a batch of 300 m-level high concrete arch dams are or will soon be under construction. Most of the dams are situated in regions of strong seismic activity. Their designed seismic accelerations reach $0.2g$ - $0.32g$ ($g = 9.81 \text{ m/s}^2$), with a probability of exceedance of 2% over a 100-year period. The designed maximum acceleration of the Dagangshan Arch Dam even reaches $0.5575g$. The influence of earthquakes should be considered and cautious arguments and investigations should be made about the seismic safety of the dams to ensure their safe operation and minimize damage from earthquakes.

Great progress has been made in the field of technology for dynamic analysis of concrete dams during earthquakes. However, some issues need to be better understood, including the nonlinear constitutive model and dynamic behavior of concrete, which are still based on elastic analysis (Lin and Chen 2001; IWHR 1997).

A rational constitutive model of concrete is the foundation of further research on the

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nonlinear behavior of concrete. An appropriate constitutive model can reflect the mechanical characteristics of the material in all the stages of deformation, including strain hardening and softening, strength reduction, and stiffness degradation, which determine the progressive damage of concrete. After several decades of development, the plastic theory describes these characteristics of concrete with a rather good theoretical basis. Nevertheless, there are too many hypotheses in the present plastic and viscoplastic models of concrete, including the Drucker postulate, associated or non-associated flow rules, and plastic potential functions, some of which have no clear physical significance and others of which do not conform to the thermodynamic laws (Leng et al. 2008). For the nonlinear analysis of concrete dams, it has become important to construct a constitutive model with few hypotheses, which can conform to the energy principle and reflect the nonlinear behavior of concrete.

In the meantime, concrete is a typical rate-sensitive material, whose strength, stiffness and ductility (or brittleness) are subject to a loading speed. Obviously, considerable deviation would be caused by the static mechanical parameters for seismic analysis. Identifying material characteristics of concrete under different strain rates and establishing proper dynamic constitutive models of concrete have become prerequisites for nonlinear dynamic analysis of arch dams. The descriptions of dynamic behavior of concrete in the *Specifications for Seismic Design of Hydraulic Structures* (IWHR 1997) are very simple and need further supplement and improvement.

Based on a consistent viscoplastic model, two thermodynamics-based consistent rate-dependent models were derived from two thermodynamics-based static models with consideration of the influence of the strain rate. They satisfy thermodynamic laws automatically and can authentically describe the dynamic behavior of concrete. With the two models, the seismic responses of the Koyuna Gravity Dam and the Dagangshan Arch Dam were analyzed. The nonlinear dynamic behavior of concrete dams and the influence of the strain rate on the dynamic behavior of concrete dams were discussed through comparison with the linear elastic model and the rate-independent models.

2 Introduction to thermodynamics-based rate-dependent constitutive models of concrete

On the basis of the classical plastic theory, a consistent rate-dependent model takes into consideration the influence of the strain rate and maintains that stress remains on the yield surface in viscoplastic flow (Wang 1997). Therefore, the consistency condition is satisfied. The yield criteria with the strain rate effect can be expressed generally as

$$f(\boldsymbol{\sigma}, \kappa, \dot{\kappa}) = 0, \quad d\lambda > 0 \quad (1)$$

where $\boldsymbol{\sigma}$ is the stress tensor, κ is an internal variable, $\dot{\kappa}$ is the rate of the internal variable, and $d\lambda$ is the viscoplastic multiplier.

Due to the rarity of multiaxial dynamic experiments of concrete at present, the multiaxial

dynamic constitutive relationship cannot be established directly. It is assumed for simplicity's sake that the dynamic increase factor (DIF) of multiaxial strength is the same as that of uniaxial strength.

Concrete shows completely different behavior under tension and under compression. Therefore, the internal variable is split into two parts, κ_c and κ_t , and behaviors under tension and under compression are described separately by means of these two internal variables. Under uniaxial compressive or tensile loading, we have

$$f_c = f(\boldsymbol{\sigma}, \kappa_c, \dot{\kappa}_c), \quad f_t = f(\boldsymbol{\sigma}, \kappa_t, \dot{\kappa}_t) \quad (2)$$

and in a complex stress state, we have

$$\kappa_c = \varphi_c(\boldsymbol{\sigma})\kappa, \quad \kappa_t = \varphi_t(\boldsymbol{\sigma})\kappa \quad (3)$$

where $\varphi_c(\boldsymbol{\sigma})$ and $\varphi_t(\boldsymbol{\sigma})$ are the weighting functions of compressive and tensile internal variables, respectively. The principles for determining $\varphi_c(\boldsymbol{\sigma})$ and $\varphi_t(\boldsymbol{\sigma})$ are as follows: for the loading process with dominant tensile stress states, $\varphi_t(\boldsymbol{\sigma})=1$ and $\varphi_c(\boldsymbol{\sigma})=0$; similarly, for the loading process with dominant compressive stress states, $\varphi_t(\boldsymbol{\sigma})=0$ and $\varphi_c(\boldsymbol{\sigma})=1$; for other loading conditions, $0 < \varphi_t(\boldsymbol{\sigma}) < 1$, $0 < \varphi_c(\boldsymbol{\sigma}) < 1$, and $\varphi_t(\boldsymbol{\sigma}) + \varphi_c(\boldsymbol{\sigma}) = 1$.

Finally, the yield criterion can be expressed as

$$f(\boldsymbol{\sigma}, \kappa, \dot{\kappa}) = f(\boldsymbol{\sigma}, \kappa_c, \kappa_t, \dot{\kappa}_c, \dot{\kappa}_t) = 0 \quad (4)$$

According to the theory of the consistent rate-dependent model, a consistency condition should be satisfied:

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} : d\boldsymbol{\sigma} + \frac{\partial f}{\partial \kappa_c} d\kappa_c + \frac{\partial f}{\partial \kappa_t} d\kappa_t + \frac{\partial f}{\partial \dot{\kappa}_c} d\dot{\kappa}_c + \frac{\partial f}{\partial \dot{\kappa}_t} d\dot{\kappa}_t = 0 \quad (5)$$

Two thermodynamics-based plastic constitutive models of concrete that have simple forms and can satisfy the energy laws naturally have been developed by Leng et al. (2008). Results from both models, obtained under static loading, agree with experimental results. The two models have been established, respectively, in Haigh-Westergaard stress space and principle stress space, the difference being that the Lode angle is neglected in the Haigh-Westergaard stress space model and considered in the principle stress space model. The two yield criteria can be expressed as

$$\frac{(p - \xi)^2}{[a_1 p + a_2(p_t - p_c)]^2} + \frac{q^2}{[b_1 p + b_2(p_t - p_c)]^2} = 1 \quad (6)$$

and

$$\frac{(\sigma_1 - \rho)^2}{A_1^2} + \frac{(\sigma_2 - \rho)^2}{A_2^2} + \frac{(\sigma_3 - \rho)^2}{A_3^2} = 1 \quad (7)$$

where p is the volumetric stress; q is the shear stress; ξ and ρ are the shift stresses on the hydrostatic axis and principle stress axis, respectively; A_1 , A_2 , and A_3 are the stress dimensions, where $A_i = a_1 \sigma_i + a_2(\sigma_{i+1} + \sigma_{i+2}) + a_3(p_t - p_c)$, ($i=1, 2, 3$); p_c and p_t are the intersection points of the yield surface and hydrostatic axis; and a_1 , a_2 , a_3 , b_1 , and b_2 are dimensionless parameters to be determined.

The plastic flow rules of the two models are, respectively,

$$\left\{ \begin{array}{l} d\varepsilon_V^p \\ d\varepsilon_S^p \end{array} \right\} = \frac{1}{\Delta} \left\{ \begin{array}{l} \frac{p - 0.5\gamma(p_t - p_c)}{A^2} \\ \frac{q}{B^2} \end{array} \right\} \quad (8)$$

and

$$d\varepsilon_1^p : d\varepsilon_2^p : d\varepsilon_3^p = [A_2^2 A_3^2 (\sigma_1 - \rho)] : [A_1^2 A_3^2 (\sigma_2 - \rho)] : [A_1^2 A_2^2 (\sigma_3 - \rho)] \quad (9)$$

where $\Delta = \sqrt{\left[\frac{p - 0.5\gamma(p_t - p_c)}{A^2} \right]^2 + \left(\frac{q}{B^2} \right)^2}$; $\gamma = \frac{2\xi}{(p_c - p_t)}$, the relative level of shift stress;

$d\varepsilon_V^p$ and $d\varepsilon_S^p$ are the volumetric plastic strain increment and deviatoric plastic strain increment, respectively; $A = a_1 p + a_2 (p_t - p_c)$; $B = b_1 p + b_2 (p_t - p_c)$; and $d\varepsilon_1^p$, $d\varepsilon_2^p$, and $d\varepsilon_3^p$ are three principle plastic strain increments.

According to the results of experiments (Suaris and Shah 1985; Reinhardt 1984), the relationships between the compressive and tensile strengths of concrete and the internal variable and rate of the internal variable are

$$f_{cd} = f_c H_c(\kappa_c) S_c(\dot{\kappa}_c) \quad (10)$$

and

$$f_{td} = f_t H_t(\kappa_t) S_t(\dot{\kappa}_t) \quad (11)$$

where f_{cd} and f_{td} are dynamic strengths of concrete under uniaxial compression and tension, respectively, and $H_c(\kappa_c)$, $S_c(\dot{\kappa}_c)$, $H_t(\kappa_t)$ and $S_t(\dot{\kappa}_t)$ are assumed functions that can be obtained from experimental data.

The internal variable is considered the equivalent plastic strain:

$$d\kappa = \sqrt{\frac{2}{3}} d\boldsymbol{\varepsilon}^{vp} : d\boldsymbol{\varepsilon}^{vp} = \sqrt{\frac{2}{3}} \boldsymbol{m} : \boldsymbol{m} d\lambda = \sqrt{\frac{2}{3}} d\lambda \quad (12)$$

where $d\boldsymbol{\varepsilon}^{vp}$ is the viscoplastic strain increment, and \boldsymbol{m} is the unit tensor of plastic flow, which is determined by Eq. (8) or Eq. (9).

Substituting these equations into the consistency condition leaves us with

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} : d\boldsymbol{\sigma} + h d\lambda + s d\dot{\lambda} = 0 \quad (13)$$

where

$$\begin{aligned} h &= a_c h_c(\kappa_c) S_c(\dot{\kappa}_c) + a_t h_t(\kappa_t) S_t(\dot{\kappa}_t) \\ s &= a_c H_c(\kappa_c) s_c(\dot{\kappa}_c) + a_t H_t(\kappa_t) s_t(\dot{\kappa}_t) \\ a_c &= \sqrt{\frac{2}{3}} \frac{\partial f}{\partial p_c} p_c \varphi_c(\boldsymbol{\sigma}) & h_c(\kappa_c) &= \frac{\partial H_c(\kappa_c)}{\partial \kappa_c} & s_c(\dot{\kappa}_c) &= \frac{\partial S_c(\dot{\kappa}_c)}{\partial \dot{\kappa}_c} \\ a_t &= \sqrt{\frac{2}{3}} \frac{\partial f}{\partial p_t} p_t \varphi_t(\boldsymbol{\sigma}) & h_t(\kappa_t) &= \frac{\partial H_t(\kappa_t)}{\partial \kappa_t} & s_t(\dot{\kappa}_t) &= \frac{\partial S_t(\dot{\kappa}_t)}{\partial \dot{\kappa}_t} \end{aligned}$$

The consistent viscoplastic model can be solved with an implicit backward Euler

algorithm, which has been studied and modified by Winnicki et al. (2001), Carosio et al. (2000) and Montans (2000).

The thermodynamic approach not only achieves greater generality, but also offers significant theoretical insights. In thermodynamics-based models, fewer hypotheses are made about the form of the dissipation function, and no assumptions are made about the plastic potential functions and flow rule. Benefitting from appropriate hardening and softening functions and relationships between the strength, modulus and strain rate, thermodynamics-based rate-dependent models reasonably describe the behavior of concrete under static and dynamic loading (Leng et al. 2008).

3 Nonlinear seismic analysis of Koyna Gravity Dam

The finite element model of the Koyna Dam, shown in Figure 1, consists of hexahedral elements, with 800 elements for the dam, 240 elements for the foundation, and a total of 2238 nodes. The material parameters for the analysis are shown in Table 1. The foundation was considered elastic material. The seismic input for the analysis was the field data of the Koyna seismic wave. The horizontal and vertical acceleration records from an accelerograph located in a gallery, with maximum accelerations of $0.49g$ and $0.34g$, respectively, were considered representative of the free-field ground motion around the dam. The normalized accelerations are shown in Figure 2. The dynamic analysis was made on the basis of the static analysis and the massless foundation model was adopted. The dam is 850 m long and 103 m high, and had a reservoir water level of 91.7 m at the time of the earthquake. Additional water mass was used to reflect the interaction between the dam and the reservoir water. Rayleigh damping was adopted and the damping ratio was 0.05.

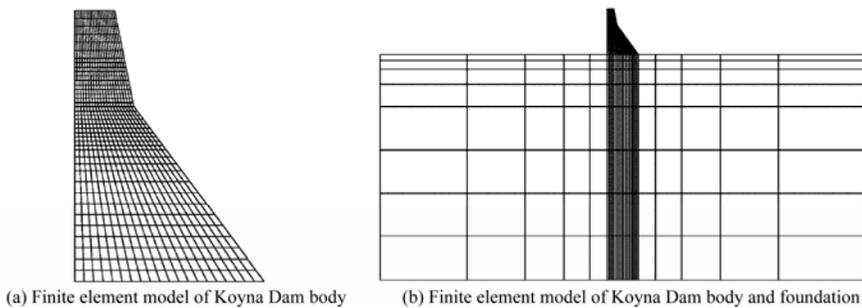


Figure 1 Finite element model of Koyna Dam

Table 1 Material properties of Koyna Dam

Dam structure	E_0 (GPa)	ν	ρ (kg/m ³)	f_c (MPa)	f_t (MPa)
Dam body	30	0.2	2 630	24	2.44
Foundation	30	0.2	2 600		

The dam was analyzed with five models: the linear elastic model, the rate-independent

model without the Lode angle, the rate-dependent model without the Lode angle, the rate-independent model with the Lode angle, and the rate-dependent model with the Lode angle. The seismic analysis results are listed in Table 2.

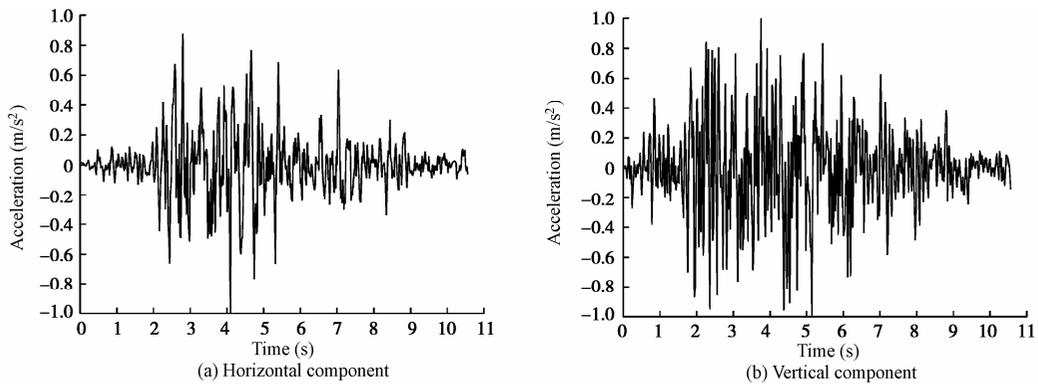


Figure 2 Earthquake ground motion components

Table 2 Results of seismic analysis of Koyna Dam

Model	Maximum first principle stress (MPa)	Maximum third principle stress (MPa)	Tensile internal variable ($\times 10^{-4}$)	Compressive internal variable ($\times 10^{-4}$)	Rate of tensile internal variable ($\times 10^{-3}$)	Rate of compressive internal variable ($\times 10^{-3}$)
Model I	7.66	11.41				
Model II	2.42	14.62	5.16	3.29		
Model III	2.72	15.10	4.78	2.54	5.56	2.65
Model IV	2.39	14.07	6.42	4.67		
Model V	2.60	14.88	4.96	3.20	5.90	3.81

The results show that the maximum first principle stresses in the five models differ considerably from one another. The first principle stress in Model I is the largest. In the other four models, the tensile stresses in the dam are redistributed substantially for strain softening, but the positions and distributions of the maximum first principle stresses remain the same. Comparisons between Model II and Model III and between Model IV and Model V demonstrate that the maximum first principle stress increases modestly when the rate effect of concrete is considered. With the redistribution of the first principle stress in the dam, the third principle stress also changes in certain ways, mainly in a varying maximum, but its distribution does not change.

The plastic strains obtained by different models under seismic loading differ from one another but show the same distribution. Plastic evolution is mainly a result of the tensile plastic strain caused by the strain softening after the concrete is cracked. Plastic deformation mainly occurs at the neck of the dam where the largest tensile stress occurs, which indicates that tensile fractures also occur in this region under seismic loading. Because the dynamic strength of concrete is higher than the static strength, the maximum plastic strain in the dam is smaller than that of the rate-independent model after the strain-rate sensitivity of concrete is

taken into account.

The distributions of the plastic strain rate in both rate-dependent models are similar to each other, except for a slight difference in value. The maximum tensile plastic strain rates in both models appear at the site of maximum plastic strain.

The maximum first principle stress and tensile internal variable from Model V are plotted in Figure 3.

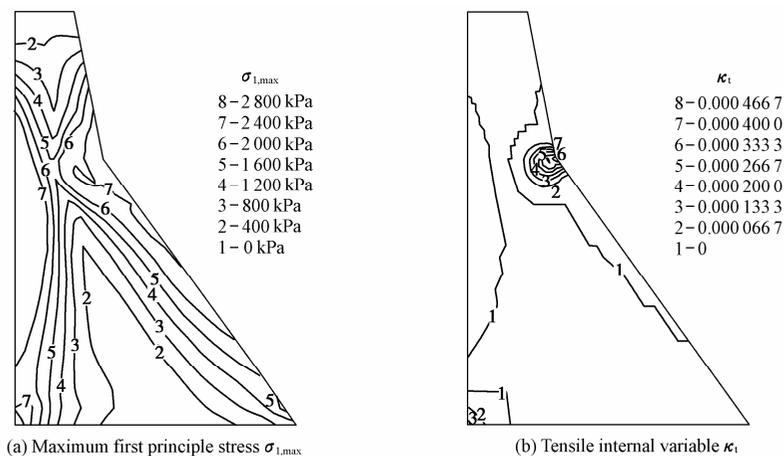


Figure 3 Maximum first principle stress and tensile internal variable in Model V

4 Nonlinear seismic analysis of Dagangshan Arch Dam

Figure 4 shows the finite element model of the Dagangshan Arch Dam. The model consists of hexahedral isoparametric elements, with 1 584 elements for the dam, 48 572 elements for the foundation, and a total of 54 087 nodes. The parameters for calculation are listed in Table 3. The foundation was assumed to be linear elastic. The static loading included the weight of the dam and the hydraulic pressure, and the influence of upriver silt was ignored. The reservoir had a depth of 195 m for the analysis. The designed seismic accelerations were 0.5575g and 0.3717g in the horizontal and vertical directions, respectively. The seismic input was the seismic wave generated by the standard response spectrum specified in the *Specifications for Seismic Design of Hydraulic Structures*. The normalized acceleration wave is shown in Figure 5. A massless foundation model and additional water mass were included for the analysis.

Table 3 Material properties of Dagangshan Arch Dam

Dam structure	E_0 (GPa)	ν	ρ (kg/m ³)	f_c (MPa)	f_t (MPa)
Dam body	24.0	0.170	2400	25	3.0
Foundation	14.5	0.258	2600		

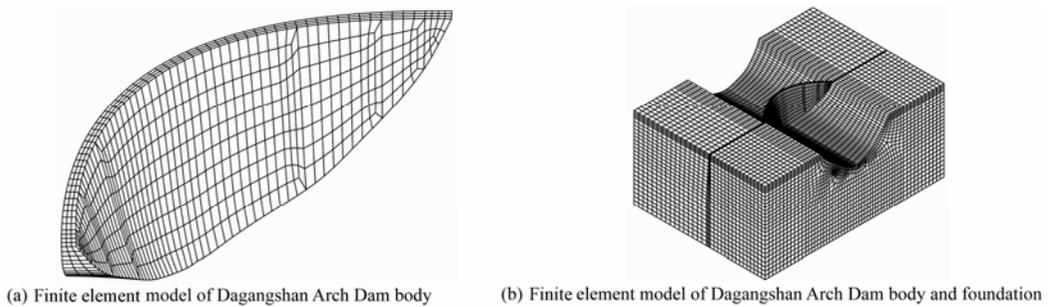


Figure 4 Finite element model of Dagangshan Arch Dam

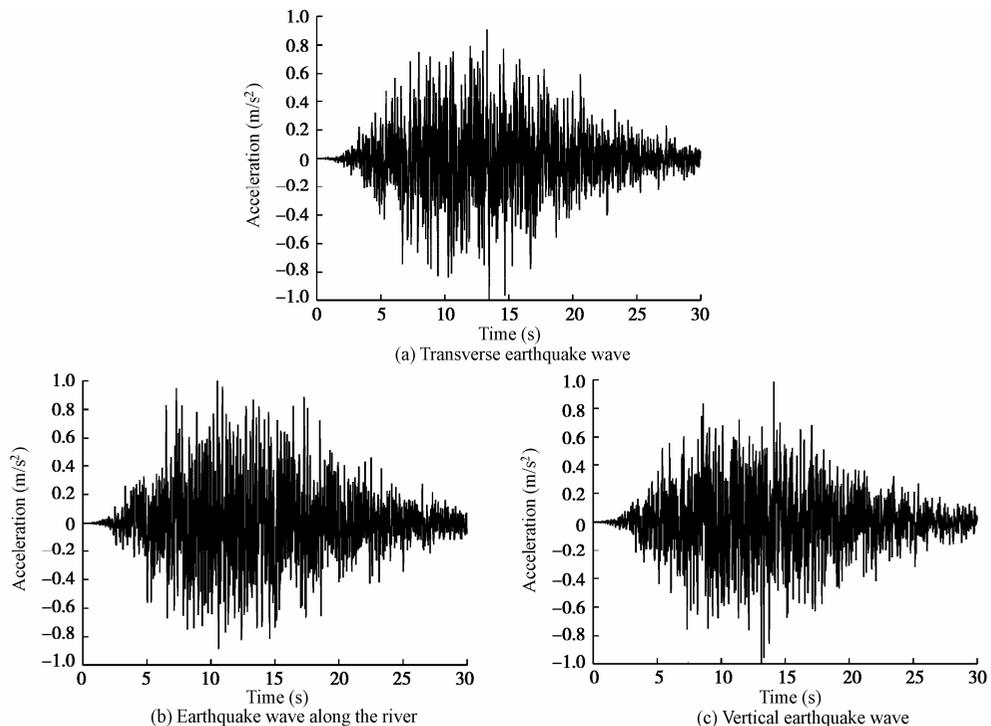


Figure 5 Earthquake wave

Table 4 shows the maximum principle stresses, tensile and compressive internal variables, and rates of internal variables in the five models. Overall, the analysis results are similar to those of the Koyna Dam. The first principle stress in the linear elastic model is the largest. The tensile stresses in the other models have been substantially redistributed. The third principle stresses have also changed with the redistributions of the first principle stresses, but the maximum values and distributions have only undergone slight changes. In the four nonlinear models, the distributions of the maximum first principle stresses are the same, and are all located in the same position, either on the bottom of the dam or on the top near the left dam shoulder. When the rate effect of concrete is considered, the maximum first principle stresses of the dam in the two plastic models increase by 9.9% and 9.2%, respectively, because the

dynamic tensile strength of concrete is grater than the static tensile strength.

The plastic zones under seismic loading obtained by different models are the same. They are mainly the zones where the tensile plastic strains occur when the cracked concrete enters the declining stage of the stress-strain curve. Under seismic loading, tensile fractures occur at the bottom of the dam and at the top near the left dam shoulder, where most of the plastic development also occurs. After consideration of the strain rate sensitivity of concrete, the comparison of various models demonstrates that the maximum plastic strain in the dam is smaller than that of the rate-independent model, and the plastic strain rate also decreases because of an increased tensile strength.

Table 4 Results of seismic analysis of Dagangshan Dam

Model	Maximum first principle stress (MPa)	Maximum third principle stress (MPa)	Tensile internal variable ($\times 10^{-3}$)	Compressive internal variable ($\times 10^{-4}$)	Rate of tensile internal variable ($\times 10^{-2}$)	Rate of compressive internal variable ($\times 10^{-3}$)
Model I	18.10	21.4				
Model II	2.92	24.4	7.75	5.87		
Model III	3.21	25.2	6.36	4.53	7.59	5.41
Model IV	2.83	23.5	7.84	6.58		
Model V	3.09	24.8	6.44	4.50	7.93	5.55

The maximum first principle stress and tensile internal variable of Model V are shown in Figure 6 and Figure 7.

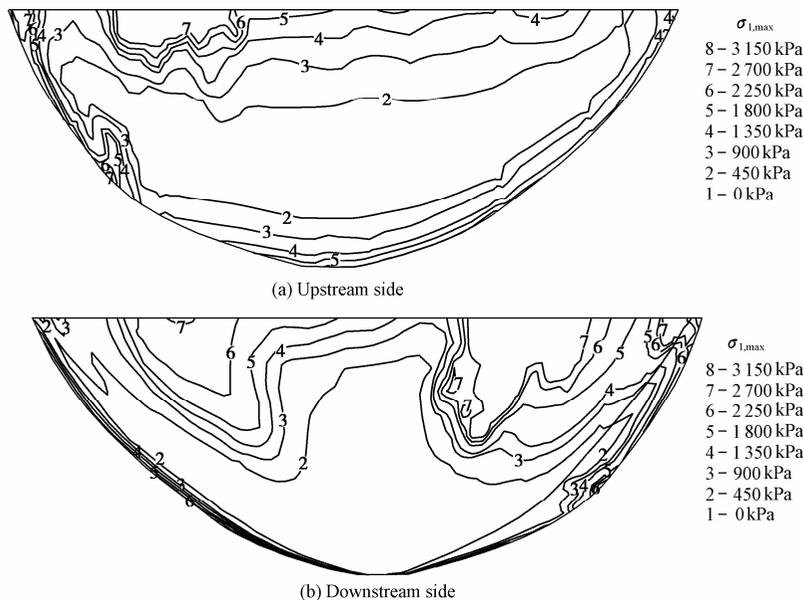


Figure 6 Maximum first principle stress $\sigma_{1,max}$ of Dagangshan Dam during an earthquake in Model V

5 Conclusions

Based on the consistent viscoplastic model and thermodynamics-based models, two

thermodynamics-based rate-dependent models of concrete were developed with consideration of the influence of the strain rate. The two models were used to perform nonlinear seismic response analysis of the Koyna Gravity Dam and Dagangshan Arch Dam. Comparison with results from a linear elastic model and two rate-independent models led to discussions of the influences of the strain rate and constitutive models on the seismic response of concrete dams. The following conclusions can be drawn from this research:

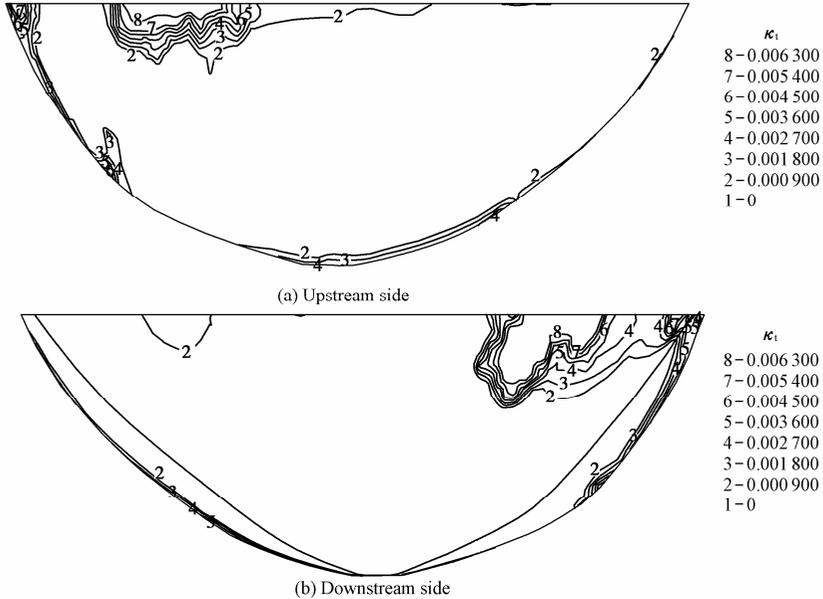


Figure 7 Tensile internal variable κ_t of Dagangshan Arch Dam during an earthquake in Model V

(1) A thermodynamics-based rate-dependent constitutive model of concrete can be derived directly from the corresponding static model, whose merits can be completely inherited. Furthermore, the rate-dependent constitutive model is convenient for calculation, capable of describing the dynamic behavior of concrete under the influence of the strain rate, and also appropriate for the nonlinear analysis of seismic response considering the strain rate sensitivity of concrete. Thermodynamics-based rate-dependent models of concrete achieve greater generality, offer significant theoretical insights, and describe the behavior of concrete more reasonably than many existing models. Nonlinear seismic analysis of concrete dams with these models can provide a better reference for design and safety verification.

(2) The compressive stress zone of concrete dams barely enters the plastic state during the seismic response. Therefore, the compressive stresses in various models show a similar value and distribution. Because the tensile stress enters the declining stage of the stress-strain curve after the concrete is cracked, the tensile stress is substantially redistributed. When the strain-rate sensitivity of concrete is considered, the tensile strength of concrete increases. Consequently, the tensile stress also increases. It is evident that, during seismic response, the

tensile stress is the control stress in the design and seismic safety evaluation of a concrete dam.

(3) In different models, the plastic strains and plastic strain rates of concrete dams show a similar distribution but vary in value. When the influence of the strain rate is considered, the maximum plastic strain increases modestly, whereas the maximum plastic strain rate decreases.

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